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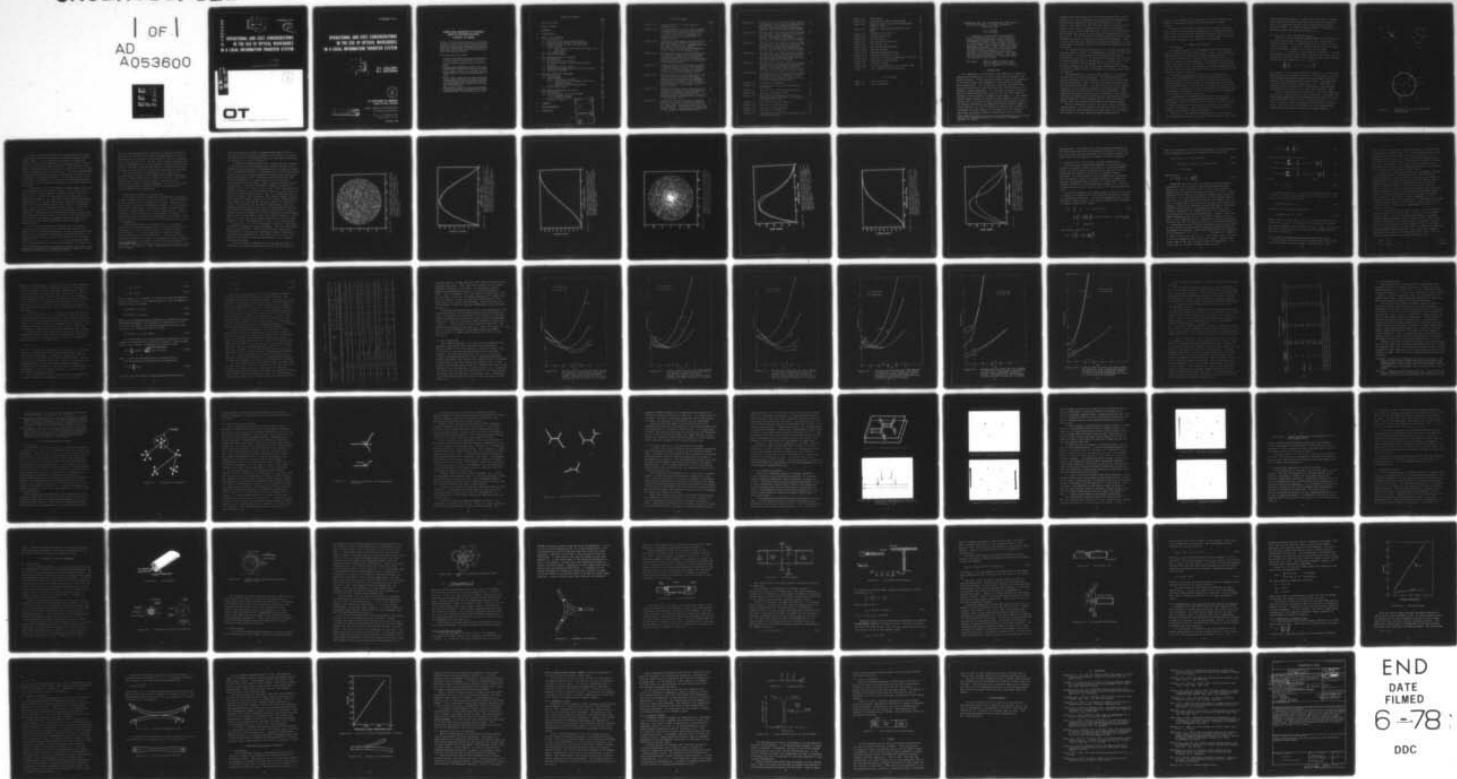
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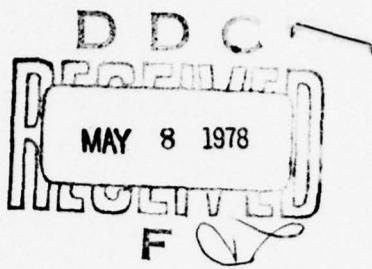
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# OPERATIONAL AND COST CONSIDERATIONS IN THE USE OF OPTICAL WAVEGUIDES IN A LOCAL INFORMATION TRANSFER SYSTEM

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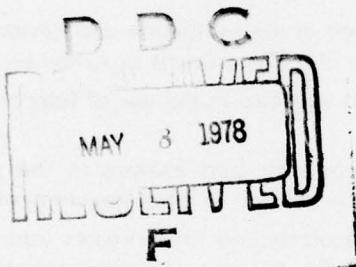
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U.S. DEPARTMENT OF COMMERCE Office of Telecommunications

# OPERATIONAL AND COST CONSIDERATIONS IN THE USE OF OPTICAL WAVEGUIDES IN A LOCAL INFORMATION TRANSFER SYSTEM



R.L. GALLAWA  
W.J. HARTMAN



U.S. DEPARTMENT OF COMMERCE  
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- Assists other government agencies in the use of telecommunications
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- Acquires, analyzes, synthesizes, and disseminates information for the efficient use of the nation's telecommunication resources.
- Performs analysis, engineering, and related administrative functions responsive to the needs of the Director of the Office of Telecommunications Policy, Executive Office of the President, in the performance of his responsibilities for the management of the radio spectrum
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OPERATIONAL AND COST CONSIDERATIONS IN THE USE OF  
OPTICAL WAVEGUIDES IN A LOCAL INFORMATION  
TRANSFER SYSTEM

R.L. Gallawa  
W.J. Hartman\*

Attention is given here to the potential use of optical waveguides in an information exchange system consisting of many terminals distributed uniformly over a circle of radius  $R$ . Cost comparison is given between coaxial cable and glass fiber systems, based on a model described in Section 2. The results show that the wide bandwidth capability of fibers leads to definite cost advantages as data rates increase. The model concentrates on the star and the loop distribution systems. The report also discusses various ways of coupling energy into and out of the fibers.

**Key words:** ARBITS, communications, data bus, economics, fiber distribution, parametric cost studies.

1. INTRODUCTION

The techniques of economically interconnecting a number of communication terminals in a typical Army Base topography, are rather involved. If one restricts attention just to the question of minimizing total line consumption, the problem is already quite complicated. If, in addition, attention is given to the realistic variation of cost with data rate, the difficulty of the problem is vastly magnified. In a realistic situation, the problem is further magnified by virtue of the fact that complete freedom in system layout is simply not available. In this regard, the Army Base Information Transfer System (ARBITS), which is of concern here, is unlike a typical intrabuilding network, which is yet another interesting and complicated multi-terminal communication problem. For the intrabuilding communication network, the topography is rather simple, in the sense that line layout is restricted according to physical obstructions such as walls,

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hallways and offices which are not normally accessible for maintenance, etc. The transmission lines normally are contained in ducts which are placed in corridors specifically for that purpose. This means that the lines are readily accessible and, hence, in the long term, cheaper, even though the resulting prolific use of cable seems wasteful. Thus, in an intrabuilding application, one cannot rely on the type of comparison given in this report. In the information transfer system of interest here (ARBITS), we permit no restrictions on system topography. This is unrealistic but the assumption allows an understanding of some of the basic principles involved in deciding an economical system topography.

In a typical ARBITS configuration, the various communication terminals serve a variety of purposes with a variety of data rate requirements. Such variability is considered in the model developed in this report. The model also considers the fact that an increased date rate can be had only at increased cost. This is quite realistic since terminal device costs increase as increasing demands are put on the speed with which it operates. In addition, cable quality must improve as increasing data rates are required.

The possibility of using optical techniques in an ARBITS configuration is an intriguing one. The characteristics of optical components are well matched to the needs of modern communication systems. Some of the key favorable characteristics are mentioned in Section 3. In addition to those listed characteristics, we call special attention to the fact that optical components are very well suited to digital techniques. Laser diodes are inherently on-off devices; furthermore, the optical waveguide can handle fast-rise-time pulses quite handily without undue distortion. This is simply not the case with coaxial cables, which are inherently well-suited for analog signals.

This is important in view of the recent trend to digital communications. At the receiver, optical detectors are quite sophisticated, having adequate speed and sensitivity to handle the demands of the most ambitious modern communication

systems. The economic model discussed in Section 2 makes allowances for these basic differences between fiber and cable systems.

Section 4 of this report concentrates on some of the practical aspects of using optical components in an ARBITS configuration. Many of the coupling components discussed there are still laboratory devices but their simplicity renders them feasible for large scale production, once the need is established.

## 2. ARBITS COST ANALYSIS

### 2.1 Introduction

An economic analysis of telecommunications facilities has become a refined and specialized area of study. Hard evidence on cost history is not often kept, except by common carriers. Even then, the normalization is often defined such that only very specific questions can be examined. Such normalization does not often lend itself to the evaluation of alternative opportunities, except in special cases. The analysis is further complicated by the fact that the telecommunications manager is not steeped in the jargon of financial matters.

An important concept in system cost is the economy of scale. In telecommunications systems, the term refers to the decreasing aggregated unit cost of a service as the number of units increases. In the context of the ARBITS program, economy of scale would apply if the cost per terminal unit decreases as more units are added to the system; alternately, it could be used to describe the decreasing cost of a unit of service (bandwidth) as the number of services increases.

The approach to the ARBITS installation costs is one of designing a least cost network configuration of acceptable performance. Thus, because there is no revenue from the system and amortization of the costs is not required, the economic picture is simplified. One needs only to concentrate on the minimum cost network, insofar as possible.

The cost of a multipoint network consists of line costs, which depend on base size, and interface and coupler costs, which

depend on system capacity. In most practical cases, a minimum cost network is obtained when total line costs are minimized. This is an important concept, first discussed by Elias and Ferguson (1974). It forms the basis of much of the effort in designing system topography for least cost.

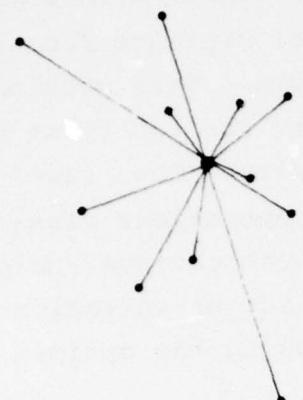
## 2.2 Star, Tee, and Loop Configurations

Three of the most important topographies in a multiterminal data bus configuration are the star, the tee, and the loop networks (Figure 2-1), so named because of the geometry of the connecting lines. In the star network (Figure 2-1a), the terminals are located at points  $(x_i, y_i)$  in the x-y plane ( $i = 1, 2, \dots, N$ , where  $N$  is the number of terminals). In order to minimize the line length (which, according to the suggestion by Elias and Ferguson (1974), will minimize system cost) one must locate the star connector at  $x_c, y_c$ , where

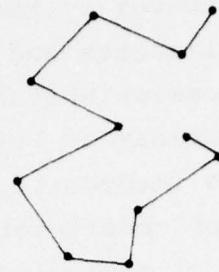
$$\sum_{i=1}^N \left[ (x_i - x_c)^2 + (y_i - y_c)^2 \right]^{1/2}$$

is minimum. Usually, the problem of finding  $x_c, y_c$  is difficult and solutions are iterative in nature. If  $N$  is large, the solution requires considerable computer storage capacity and long computer runs (Nesenberg and Linfield, 1976). Approximations are often used to obtain values of  $(x_c, y_c)$ . The reader is referred to the literature for a discussion of the solutions (Nesenberg and Linfield, 1976; Kuhn, 1965; and Katz, 1969).

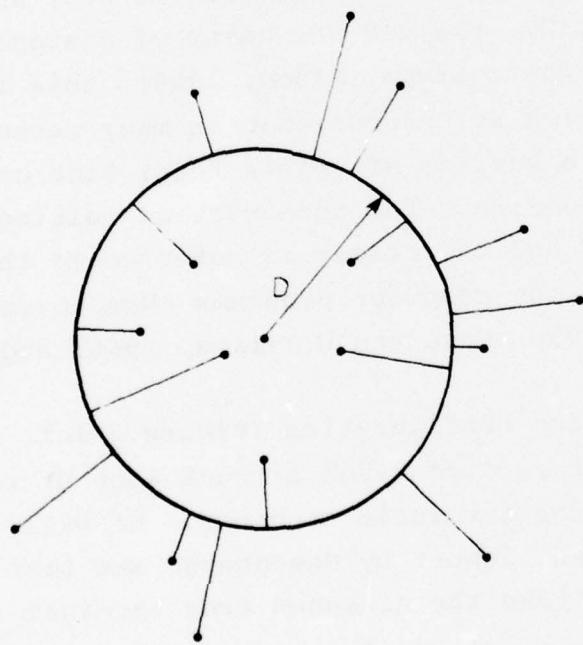
Several approximations are available to find the values  $(x_c, y_c)$  since the exact problem is quite difficult. If we make the simplifying assumption that the communications nodes are distributed uniformly within a circle of radius  $R$ , then  $(x_c, y_c)$  is  $(0, 0)$ . Specific types of uniform distribution are discussed below. The nature of the distribution of the communication terminals has a significant impact on network topology.



(a)



(b)



(c)

Figure 2-1. The star (a), tee (b), and loop (c) configurations.

For the tee system (Figure 2-1b) minimum total line length can be had by solving the so-called traveling-salesman problem (Flood, 1956). In this problem, we consider a salesman who wishes to travel by the shortest total distance from his home to each of  $N-1$  cities and then return home. This problem is attributed to Hassler Whitney, who posed it for the first time in 1934 during a seminar at Princeton University. The traveling-salesman problem has interesting analogies in operations research and is therefore of considerable interest. One obvious analogy is the scheduling of a machine over a given set of operations when the cycle time is to be minimized by choosing the optimum sequence of operations.

One approach is for the traveling salesman always to go next to the closest city not already visited. In an actual test of this attempt at least distance, consideration was given to 49 cities, one in each of the contiguous United States and the District of Columbia. The next-closest-city approach, beginning in Washington, DC, yielded 904 units of distance as opposed to 499 units for the optimum (Flood, 1956); this is an increase of nearly 30%, which is unacceptable in many cases. In data bus applications, a savings of 30% in total line used can be important. Algorithms for constructing multipoint networks have been developed and the reader is referred to the literature for a description of the computer programs (Nesenberg and Linfield, 1976; Flood, 1956; Easu and Williams, 1966; and Chandy and Russell, 1972).

For the loop configuration (Figure 2-1c), we consider a service area of radius\*  $R$  and a trunk loop of radius  $D$ , onto which each of the terminals connects. We will follow many of the ideas of a recent report by Nesenberg and Linfield (1976). In particular, we take the distance from terminal  $i$  to the loop of radius  $D$  as  $x_i$ ,  $i = 1, 2, 3, \dots, N$ . In the special case when  $D=0$ , the loop reduces to the star configuration. The loop and

\*In a later section we use  $R_s$  and  $R_D$  to designate bit rates. The service area radius ( $R$ ) is designated without a subscript. That should allow a distinction without confusion.

the star configurations have a potential advantage over the tee since not all lines carry the same (maximum) traffic; in fact, the drop lines or the lines from the node to the star carry only the traffic appropriate to the node. The traffic carried in the loop is comparable to the traffic carried in each line in the tee network. For that reason we now restrict attention to the star and loop configurations to examine cost variations in each case.

The star and loop configurations can be treated in a straightforward fashion if the communication nodes are distributed in a known fashion over a circle of radius  $R$ . The actual distribution of such nodes will play an important part in choosing an economical system topography. It is for this reason that we digress now to consider realistic alternatives in a typical ARBITS nodal distribution.

### 2.3 Communication Terminal Distributions

We consider a base containing many communication terminals (nodes) distributed over a circle of radius  $R$ . The distribution of the terminals will be assumed to be random. Normally, one would expect a central or headquarters facility to be located at the center of the circle. The other terminals are then distributed in a random fashion with a density which may be a function of distance from the center ( $r$ )\* but constant in azimuth. In some cases, there is no special significance in the center of the circle ( $r=0$ ) and the nodal density is uniform with respect to the  $x$ - $y$  coordinates.

A distribution which is uniform in the  $r$ ,  $\theta$  coordinate system will be densely populated near  $r=0$  and the density will decrease as  $1/r$  since the area in the  $r$ ,  $\theta$  system increases linearly with  $r$ . It is also possible to consider a density function which varies arbitrarily with  $r$ . The distribution is important inasmuch as it dictate the probability density function for distance between terminals. We will restrict attention to

\*The symbol  $r$  denotes the variable distance from the center of the circle;  $R$  is the limiting radius, within which the terminals are all contained.

the distribution of terminals (communication nodes) which is uniform in the rectangular coordinates (but limited to a circle of radius  $R$ ) and a distribution uniform in the polar coordinates and limited to a circle of radius  $R$ .

A computer program written by R.K. Rosich and his colleagues at the Institute for Telecommunication Sciences (Rosich, 1977), considers the distribution of random points in a circle of radius  $R=1$ , distributed uniformly in rectangular and in polar coordinates. Figure 2-2a gives a computer-generated scatter-point display of such random points in the rectangular coordinate system. Note that the probability of finding a point in an area  $dxdy$  is proportional to the area  $dxdy$ . (This is what we mean by uniform distribution.) The points represent a computer selection of random points in the circle of unit radius. After the points are randomly generated, a bar-histogram of 100 cells is generated to show the distance between points. This is a probability distribution function of such distances. The result is shown in Figure 2-2b, which also shows (smooth curve) the analytic description of this probability distribution function (Crow, 1972, Equation 5). Figure 2-2c, included for the sake of completeness, shows the computer generated (stair-step) curve and the analytic curve for the cumulative probability distribution function for a distance between two points appropriate to the distribution of Figure 2-2a. These curves are included to assist the reader in making a judgment on the relevance of the model presented here to the actual problem at hand. The population of Figure 2-2a is 5000 points.

Figures 2-3a, b, c are corresponding figures for 5000 points uniformly distributed in polar coordinates. Note the crowding of points near the origin. This may realistically describe the distribution in some Army Base networks for which the point  $r=0$  represents base headquarters. The smooth curve in Figure 2-3b shows the Beta density function approximation to the computer generated histogram.

Figure 2-4 shows a comparison of the probability density functions of distance between two points for the two types of

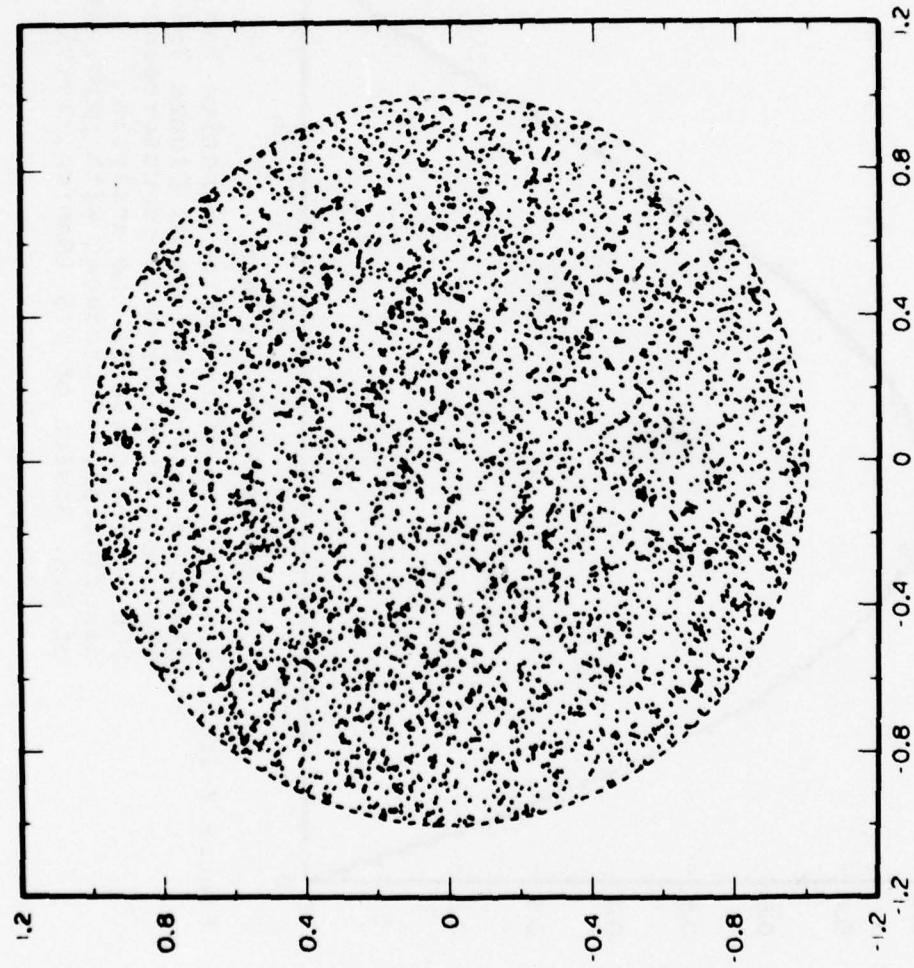


Figure 2-2a. Computer-generated scatter-point display of random points uniformly distributed in a rectangular coordinate system, in a circle of radius  $R=1$ . Reproduced with permission of R.K. Rosich of ITS (Rosich, 1977).

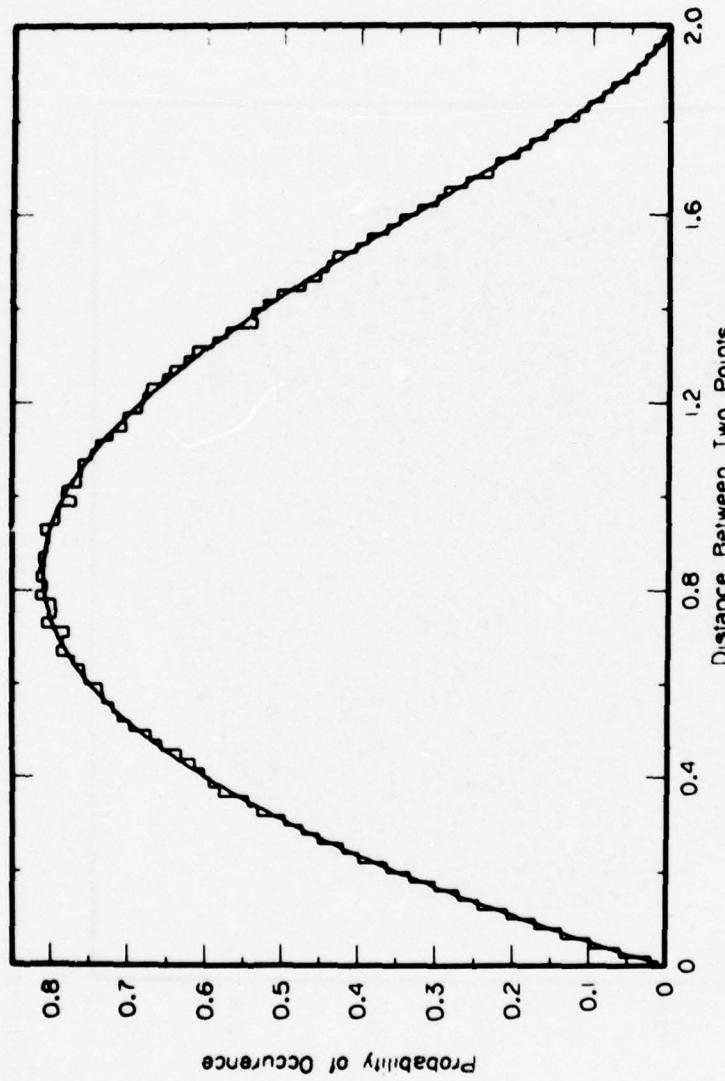


Figure 2-2b.

Probability distribution function for the distance between points of Figure 2-2a. The stair-step curve is computer-generated; the smooth curve is the analytic description. Reproduced with permission of R.K. Rosich of ITS (Rosich, 1977).

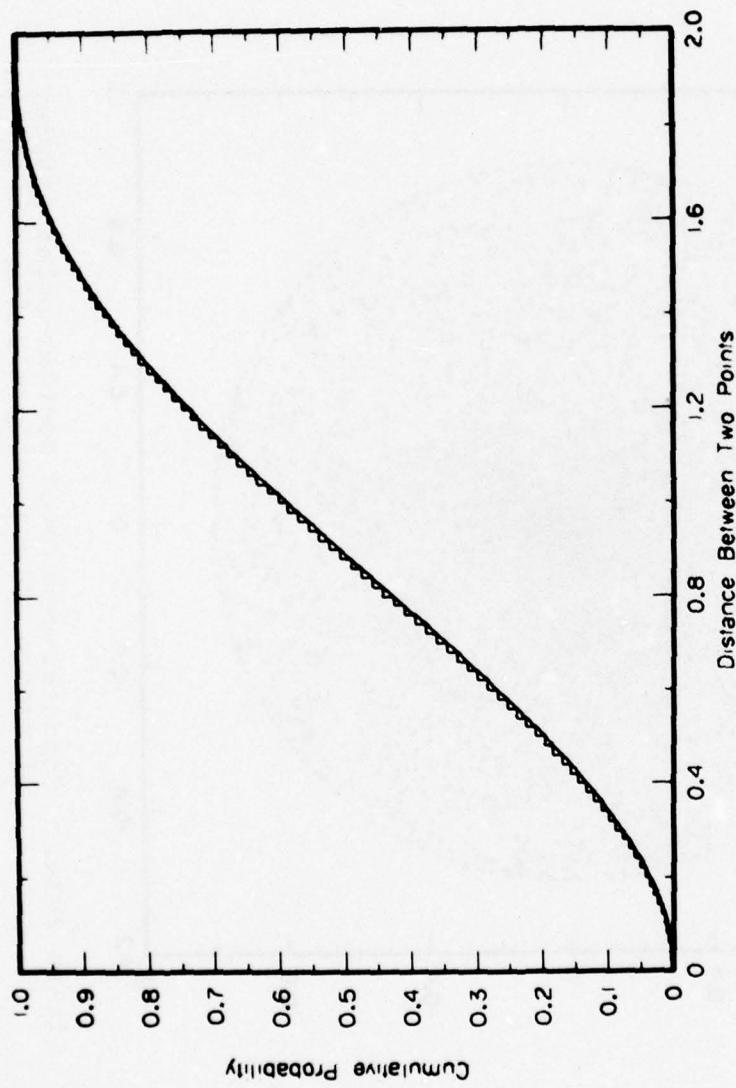


Figure 2-2c.

Computer-generated (stair-step curve) and the analytic (smooth curve) description of cumulative probability function for distance between two points of Figure 2-2a. Reproduced with permission of R.K. Rosich of ITS (Rosich, 1977).

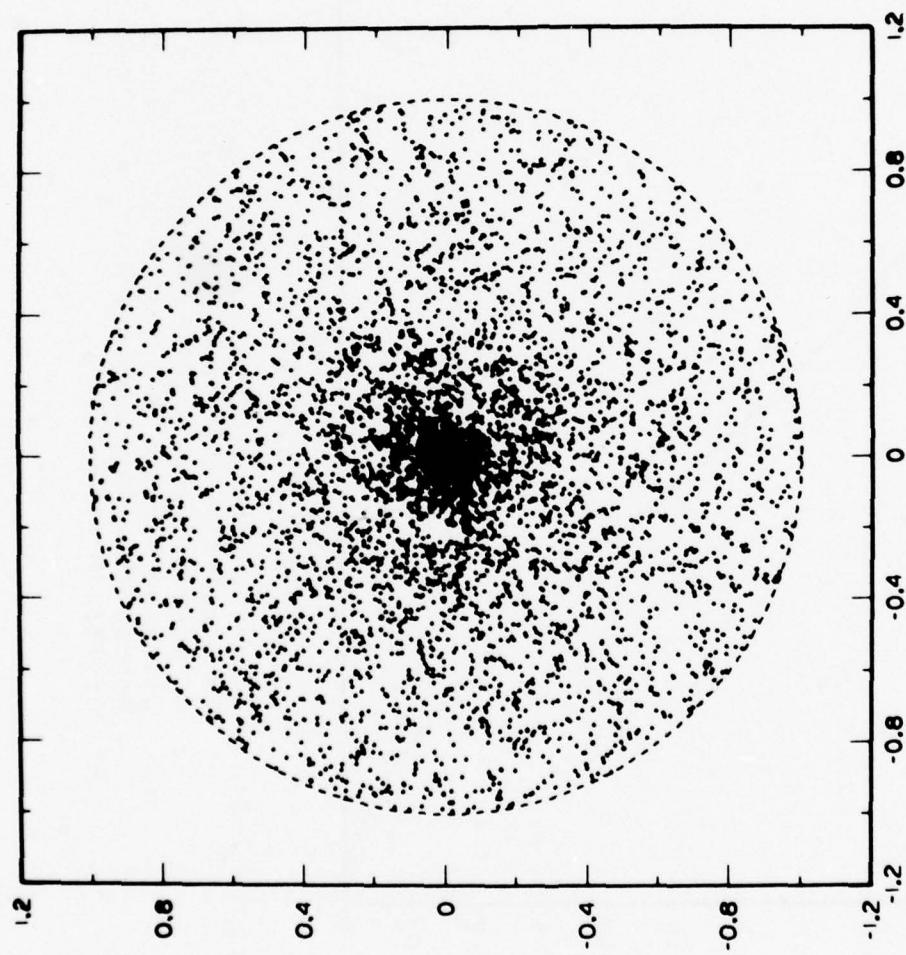


Figure 2-3a. Computer-generated scatter-point display of random points uniformly distributed in a polar coordinate system, in a circle of radius  $R=1$ . Reproduced with permission of R.K. Rosich of ITS (Rosich, 1977).

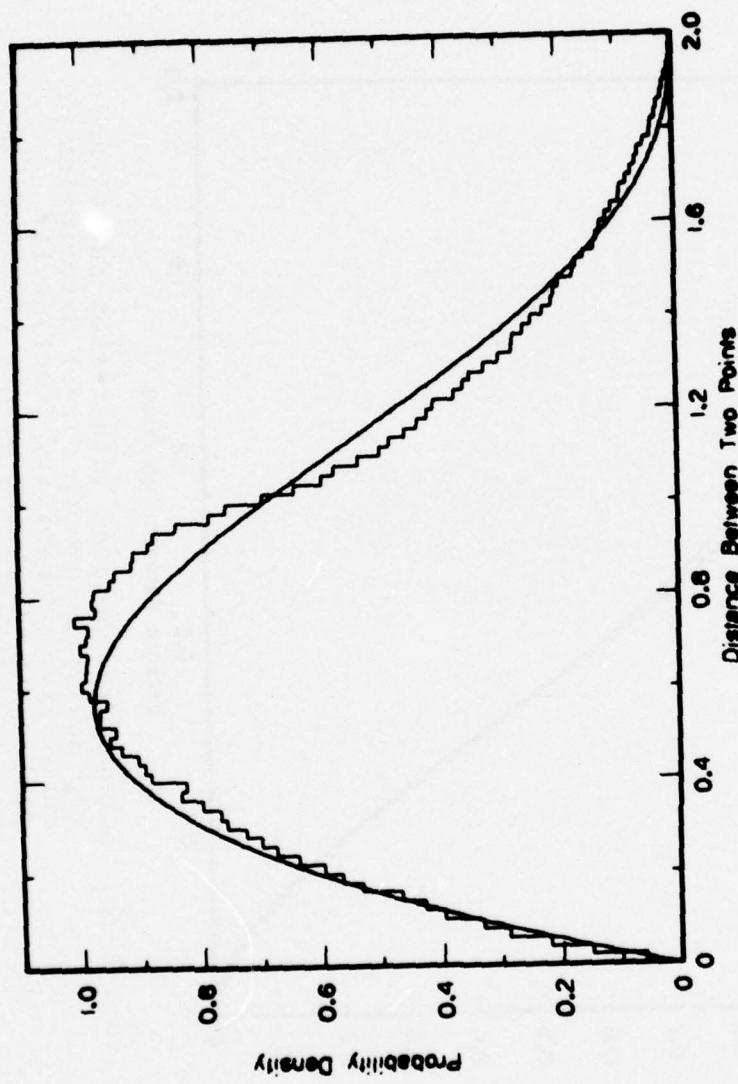


Figure 2-3b. Computer-generated (stair-step curve) and the Beta density function approximate (smooth curve) description of probability distribution function for the distance between points of Figure 2-3a. Reproduced with permission of R.K. Rosich of ITS (Rosich, 1977).

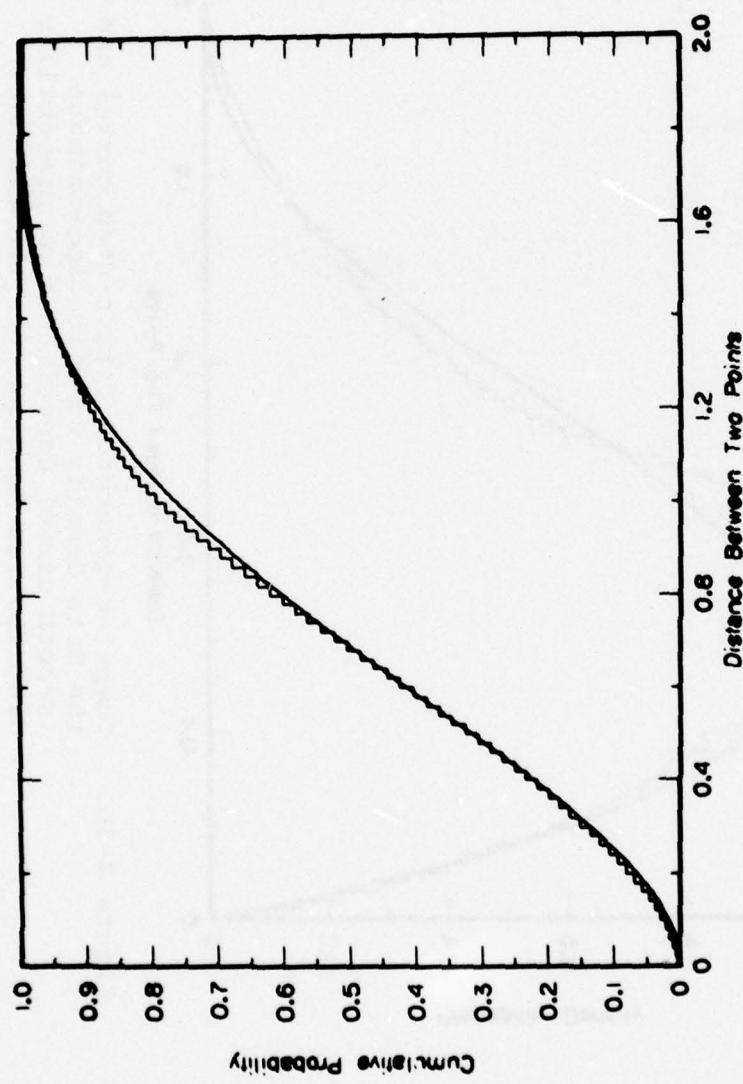


Figure 2-3c.

Computer-generated (stair-step curve) and the analytic (smooth curve) description of cumulative probability function for distance between two points of Figure 2-3a. Reproduced with permission of R.K. Rosich of ITS (Rosich, 1977).

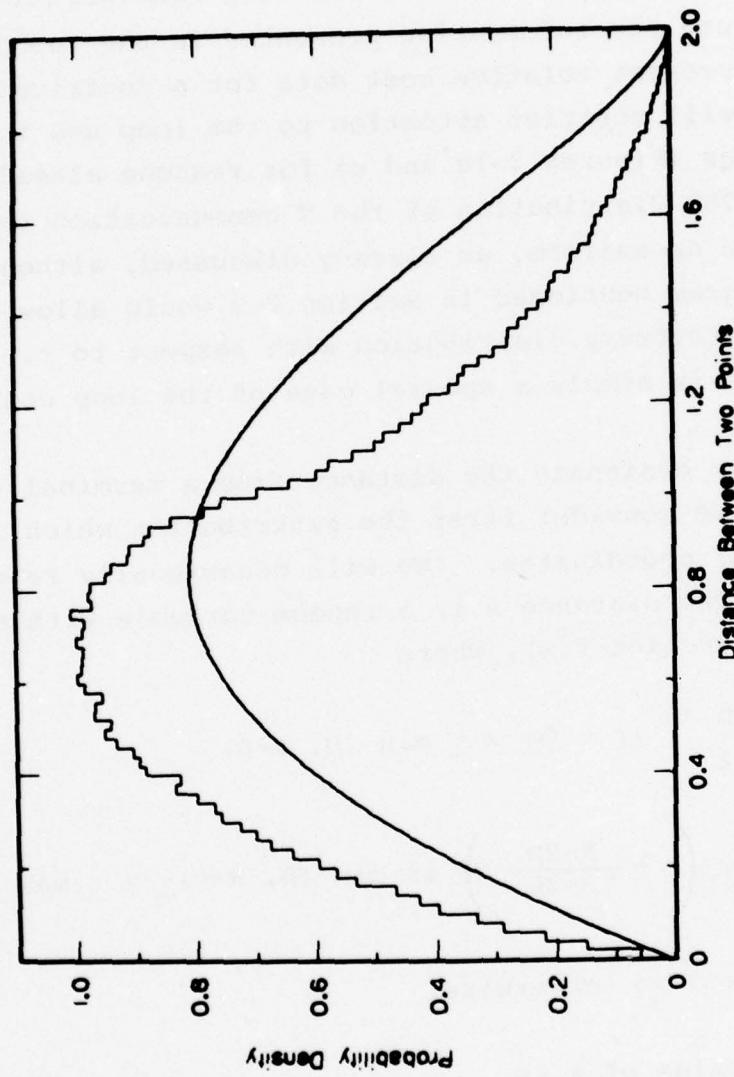


Figure 2-4.

Probability density function of distance between two points for polar distribution of Figure 2-3a (stair-step curve) and for the rectangular distribution of Figure 2-2a (smooth curve). Reproduced with permission of R.K. Rosich of ITS (Rosich, 1977).

distributions. The smooth curve is the analytic expression for the distribution uniform in rectangular coordinates. The stair-step curve shows the bar histogram for the distance between two points when the distribution is uniform in polar coordinates.

#### 2.4 Relative Costs for the Star and Loop Configurations

We now use the information presented in the two preceding sections to present relative cost data for a contrived ARBITS system. We will restrict attention to the loop and the star configurations (Figures 2-1a and c) for reasons already described. The distribution of the  $N$  communication terminals will be taken as uniform, as already discussed, although the computer program mentioned in Section 2.3 would allow one to consider an arbitrary distribution with respect to  $r$ . The star configuration is simply a special case of the loop configuration ( $D=0$ ).

Letting  $x$  designate the distance from a terminal to the loop of radius  $D$ , we consider first the distribution which is uniform in rectangular coordinates. (We will occasionally refer to this as Case A.) The distance  $x$  is a random variable with a probability density function  $f(x)$ , where

$$f(x) = \frac{4D}{R^2} \quad \text{if} \quad 0 \leq x \leq \min(D, R-D) \quad (2-1a)$$

$$= \frac{2}{R^2} \left( r + \frac{R-2D}{|R-2D|} x \right) \quad \text{if} \quad \min(D, R-D) \leq x \leq \max(D, R-D), \quad (2-1b)$$

$$= 0 \quad \text{otherwise.}$$

The expected value of  $x$  is

$$Ex = \frac{2}{3} R \left[ 1 - \frac{3}{2} \frac{D}{R} + \left( \frac{D}{R} \right)^3 \right] . \quad (2-2)$$

When the  $N$  terminals are distributed uniformly in polar coordinates the corresponding probability density function is (Case B)

$$f(x) = \frac{2}{R} \text{ if } 0 \leq x \leq \min(D, R-D), \quad (2-3a)$$

$$= \frac{1}{R} \text{ if } \min(D, R-D) \leq x \leq \max(D, R-D), \quad (2-3b)$$

$$= 0 \text{ otherwise,}$$

and (Case B)

$$Ex = \frac{R}{2} \left[ 1 - 2 \frac{D}{R} + 2 \left( \frac{D}{R} \right)^2 \right]. \quad (2-4)$$

Equations (2-2) and (2-4) are amenable to an interesting comparison of the two types of terminal distributions being considered. Case A (Equation (2-2), Figure 2-2a) exhibits a minimum value of expected distance,  $Ex$ , when  $D/R = 1/\sqrt{2}$ . Case B (Equation (2-3), Figure 2-3a) exhibits minimum value of  $Ex$  when  $D/R = 1/2$ . It is not surprising that the distribution which is uniform in  $r-\theta$  coordinates has minimum  $Ex$  at a smaller value of  $D/R$ . In either case, the minimum cost may not occur when  $Ex$  is minimum. This will be discussed further when data are presented. Choosing  $D/R$  such that  $Ex$  is minimum has operational (non-economic) advantage in an ARBITS system so it is desirable to have minimum  $Ex$  coincide with minimum cost. Whether or not this happens depends on the relative importance of cable or fiber cost. Total transmission line consumption is minimum when  $D/R$  is slightly less (by  $\pi/N$ ) than the value of  $D/R$  yielding minimum  $Ex$ .

Following the earlier work of Nesenbergs and Linfield (1976), we now consider various scenarios involving different services and number of terminals. The various services will be identified by the subscript  $s$ ,  $s = 1, 2, \dots$ . The number of terminals of type  $s$  will be  $n_s$ , and  $\sum n_s = N$ . Each feeder line, of length  $x$ , will be assumed to cost  $\lambda_s$ , per unit length. The per-unit-length cost of the loop of radius  $D$  will be designated  $\lambda$ . The total cost,  $Y$ , is then a random variable given by

$$Y = 2\pi D \lambda + \sum_s n_s \sum_{i=1}^{n_s} x_i , \quad (2-5)$$

and the expected cost for Case A is

$$EY(A) = \frac{2}{3} R \sum_s n_s \lambda_s \left[ 1 - \frac{3}{2} \frac{D}{R} (2\alpha^2) + \left(\frac{D}{R}\right)^3 \right]. \quad (2-6)$$

For Case B, the expected cost is

$$EY(B) = \frac{R}{2} \sum_s n_s \lambda_s \left[ 1 - 2 \frac{D}{R} (2\alpha^2) + 2 \left(\frac{D}{R}\right)^2 \right], \quad (2-7)$$

where

$$2\alpha^2 = 1 - \frac{2\pi\lambda}{\sum_s n_s \lambda_s} . \quad (2-8)$$

Equations (2-6) and (2-7) can be differentiated with respect to  $(D/R)$  to determine the optimum value of loop radius,  $D$ . The least cost for Case A occurs when

$$D \text{ (optimum, Case A)} = \alpha R. \quad (2-9)$$

For Case B, the least cost occurs when

$$D \text{ (optimum, Case B)} = \alpha^2 R. \quad (2-10)$$

Notice that if the unit loop cost  $(2\pi\lambda)$  is greater than the unit feeder cost  $(\sum_s n_s \lambda_s)$ ,  $\alpha$  is imaginary ( $\alpha^2$  is negative), meaning that the star network ( $D=0$ ) is the least cost. When  $\alpha$  is positive and real, Equations (2-9) and (2-10) give the optimum radius of the loop in each case.

#### 2.4.1 Unit cost and data rates for cable and fiber lines

The economic model presented here compares the relative cost of metallic cable and optical fiber. Most generally, the

metallic cable is expected to be coaxial cable; the model is not so restricted, although the values of the constants used in this report apply to coaxial cables. We will later make a key assumption regarding the fact that, for metallic cables, unit cost increases in proportion to the product  $B \cdot L$  where  $B$  is pulse rate and  $L$  is distance. This concept is felt to hold for all metallic cables, although the multiplying constant of proportionality changes with the geometry of the cable.

We immediately encounter a problem in terminology. We want to distinguish metallic transmission lines from glass fiber transmission lines. We will do so by using the term "cable" or "coaxial cable" to identify the metallic line, with the understanding that it is only the constants used that limit the results to coaxial cables. We will use the term "fiber" or "optical fiber" or some such term to distinguish the glass fiber optical transmission line. The problem in terminology arises because the fiber waveguides are, in every sense, a "cable". Thus, to use the term "cable" to mean a metallic line is a misnomer, common usage notwithstanding. In the interest of ease of presentation, however, we will use the term "cable" to mean a metallic cable. Glass fiber transmission lines will be identified as fibers.

It is generally conceded that the unit cost of communication services depends on the data rate. In fact, it can be argued that for cables, the unit cost will also depend on distance between stations (Chadwick, 1977; Gallawa, 1977). This can be seen by noting that, for coaxial cable, a reasonable figure of merit for digital traffic is the product  $BL^2$  where  $B$  is pulse rate and  $L$  is distance. In fact,  $BL^2$  is a constant in dispersion limited coaxial cable systems with the value of the constant depending on the amount of copper used in the cable. This is seen by noting that for such cables (Gallawa, 1977)

$$BL^2 = K\pi a^2 \quad (2-11a)$$

$$BL = K\pi a^2/L \quad (2-11b)$$

where  $K$  is a constant and  $a$  is the radius of the inner conductor;  $K$  depends on characteristic impedance, geometry, conductivity, and the acceptable level of intersymbol interference. Thus, if  $L$  is to increase for fixed pulse rate, the area of the inner conductor must increase in direct proportion. The right-hand side of Equation (2-11) suggests that the cost, per unit length, of the cable varies in proportion to the product of  $B$  and  $L$ . Equation (2-11) does not take signal equalization into account but such accounting would only serve to increase the value of the constant,  $K$ .

From Equation (2-11), it is logical to assume that the unit cost of service will increase in proportion to the line length required, provided that coaxial cable is used and that the system is operating in the dispersion limited regime. The latter contingency is actually a generous assumption since operation in the attenuation limited regime is more restrictive; in that case, degradation is exponential (rather than algebraic) with distance.

If optical glass fiber waveguide is used, the unit cost of communication services will likely be independent of the length of line used. This is because the expression corresponding to Equation (2-11a) is (Gallawa, 1976)

$$BL^{\gamma} = K \quad (2-12)$$

where  $K'$  is a constant, independent of fiber size;  $\gamma$  is between  $1/4$  and  $1$ , depending on mode-mixing and whether the fiber is single mode or multimode. Comparison of Equations (2-11) and (2-12) reveals the advantage of optical fiber systems: the size of the optical waveguide (and, hence, the cost of purchase and installation) need not be increased in order to improve its length-dependent capability. Thus, the cost, per unit length, is independent of distance between terminals. We admit a cost dependence on pulse rate, however.

The aforementioned factors are accounted for by making the following assumptions, appropriate to the use of cables:

$$\lambda = C_0 + C_1' R_D \quad (2-13)$$

$$\lambda_s = C_0 + C_1'' R_s \quad (2-14)$$

where  $C_0$  and  $C_1$  are constants,  $R_s$  is the data rate for service  $s$  and  $R_D$  is data rate on the loop of radius  $D$ ;  $C_1'$  and  $C_1''$  depend on distance and  $C_1$  as follows:

$$C_1' \text{ (cable)} = C_1 D/r_0 \quad (2-15)$$

$$C_1'' \text{ (cable)} = C_1 E_x/r_0 \quad (2-16)$$

where  $r_0$  is a reference value of distance, introduced to maintain the proper dimensions in the equations. Equations (2-15) and (2-16) are for cables. For optical waveguide systems, we take  $C_1'$  and  $C_1''$  to be independent of line length.

$$C_1' \text{ (fiber)} = C_1 = C_1'' \text{ (fiber)} \quad (2-17)$$

If we now take  $p_s$  to be the occupancy (probability of transmission or reception) for a terminal of service  $s$ , then the mean plus  $4\sigma$  value of  $R_D$  is (Nesenberg and Linfield, 1976)

$$R_D = \sum_s n_s p_s R_s + 4 \sqrt{\sum_s n_s p_s (1-p_s) R_s^2} \quad (2-18)$$

where  $\sigma$  is the standard deviation of the distribution.

Let the average terminal activity be denoted by  $p$ :

$$p = \frac{1}{N} \sum_s n_s p_s; \quad (2-19)$$

$x_s$  and  $y_s$  are the relative terminal population and activity:

$$x_s = n_s/N \quad (2-20)$$

$$y_s = p_s/p \quad . \quad (2-21)$$

With these definitions, we are in a position to compare the cable and the fiber systems for various scenarios. The scenarios are defined by specifying values of the  $x_s$ ,  $y_s$ ,  $R_s$ ,  $p$ ,  $N$ , and the ratio  $C_o/C_1$ , and the ratio  $R/r_o$ . Table 2.1 gives the details of three scenarios of varying activity. For Scenario 1, 50% of all terminals are voice ( $x_s = 0.50$ ), while the second scenario is all voice. Scenario 3 is rather ambitious, with 8% of its terminals requiring data rates in excess of 1 MBPS; 1% of the terminals are 33 MBPS video terminals ( $x_s = 0.01$ ). Recall that the value assigned to  $y_s$  specifies the activity of that terminal, with respect to the system average:  $y_s = 0.6$  means that the relative activity for a telephone (Scenario 1) is 60% of the average terminal activity over the entire network.

In the data presented below, we will consider a system having 400 terminals ( $=N$ ) and activity ( $=p$ ) of 0.25 and 0.75. The ratio  $C_o/C_1$  for the cable will be taken to be 150 with data rate given in KBPS. This means that the data rate begins to impact on cost (Equations (2-13) and (2-14)) when the data rate approaches 150 KBPS if (for the cable)  $D$  or  $Ex$  is equal to the reference value  $r_o$  (see Equations (2-15) and (2-16)). We will take  $R/r_o = 4$ , where  $R$  is the radius of the service area, as previously defined. This means that if characteristic distance ( $D$  or  $Ex$ ) is less than about  $R/4$ , distance will have little impact on the per-unit-length cost; as characteristic distance approaches  $R/4$ , however, the unit cost for cable will increase.

For the data presented in the curves, below, we assume that the fiber base cost (reflected ultimately in the value of  $C_o$ ) is 1.5 times the cable base cost. Thus, while the fiber has distinct operational advantages over cable, the current unit cost is definitely higher than cable unit cost. For the high data rate systems, however, it is expected that the normalized cost of

Table 2.1. Three Scenarios

| Service Index<br>s | Service Type                              | Rate<br>$R_s$ | Scenario 1 |       | Scenario 2 |       | Scenario 3 |       |
|--------------------|---|---------------|------------|-------|------------|-------|------------|-------|
|                    |   |               | $x_s$      | $y_s$ | $x_s$      | $y_s$ | $x_s$      | $y_s$ |
| 1                  | Data                                      | 2.4 Kbps      | 0.20       | 1.5   | -          | -     | 0.20       | 1.0   |
| 2                  | Data                                      | 4.8 Kbps      | 0.10       | 2.0   | -          | -     | 0.10       | 1.0   |
| 3                  | Data                                      | 9.6 Kbps      | 0.05       | 2.0   | -          | -     | 0.10       | 0.5   |
| 4                  | Low Speed Fax                             |               | 0.05       | 1.5   | -          | -     | 0.10       | 3.0   |
| 5                  | Voice                                     | 16.0 Kbps     | 0.50       | 0.6   | 1.0        | 1.0   | 0.20       | 1.0   |
| 6                  | High Speed Fax                            | 56.0 Kbps     | 0.05       | 0.1   | -          | -     | 0.02       | 0.5   |
| 7                  | High Speed Data                           |               | 0.05       | 0.4   | -          | -     | 0.20       | 0.3   |
| 8                  | Color Fax                                 | 1.344 Mbps    | 0          | -     | -          | -     | 0.02       | 0.5   |
| 9                  | Slow Scan Video                           |               | 0          | -     | -          | -     | 0.02       | 2.5   |
| 10                 | Picture-phone Video                       | 3.1 Mbps      | 0          | -     | -          | -     | 0.02       | 0.5   |
| 11                 | NTSC Video<br>(Coded, High Cost<br>Term.) | 14.0 Mbps     | 0          | -     | -          | -     | 0.01       | 0.4   |
| 12                 | NTSC Video<br>(Coded, Low Cost<br>Term.)  | 33.0 Mbps     | 0          | -     | -          | -     | 0.01       | 0.6   |

the fiber systems is lower than the cable system in spite of the increased base price. This comes about because of the extreme bandwidth capability of the fiber. It can support tens of megabits per second easily even in a relatively inexpensive fiber. This is accounted for in the model by taking  $C_0/C_1 = 450$  for the fiber. Even this is probably pessimistic; it means that data rate impacts on cost for the fiber system only for data rates close to or greater than 450 KBPS. Below that data rate, the unit cost of fiber is flat and independent of data rate and distance,  $L$ .

In the data presented below, we will give dimensionless cost (normalized cost); with the normalization being with respect to  $r_0 \cdot C_0$ . Furthermore, for simplicity, we will present "universal" curves by normalizing each to unity with respect to the maximum fiber cost. Thus, the curves for fiber systems are always unity or less. All other values are with respect to that cost. The normalization depends on the value of  $p$ ; thus, even though  $p = 0.25$  and  $p = 0.75$  are presented in the same figure, the reader must be careful to compare cable and fiber only for the same value of  $p$ .

## 2.5 Cost Comparisons

The curves in Figures 2-5 to 2-10 give the results of the analysis, based on the model discussed in the preceding. Figures 2-5 and 2-6 are for Scenario 1, Figures 2-7 and 2-8 are for Scenario 2, and the last two figures are for the last scenario. The odd numbered figures (2-5, 2-7, 2-9) are for terminal distributions uniform in the rectangular coordinate system (see Figure 2-2a) and the even numbered figures correspond to uniformity in the polar system (see Figure 2-3a). A mark appearing on the abscissa in each case indicates the value of the abscissa (D/R) corresponding to minimum value of  $E_x$ . As discussed earlier, this is also close to the value of D/R corresponding to minimum total consumption of transmission line. Comparison can be made between cable and fiber in each case only for like values of  $p$ .

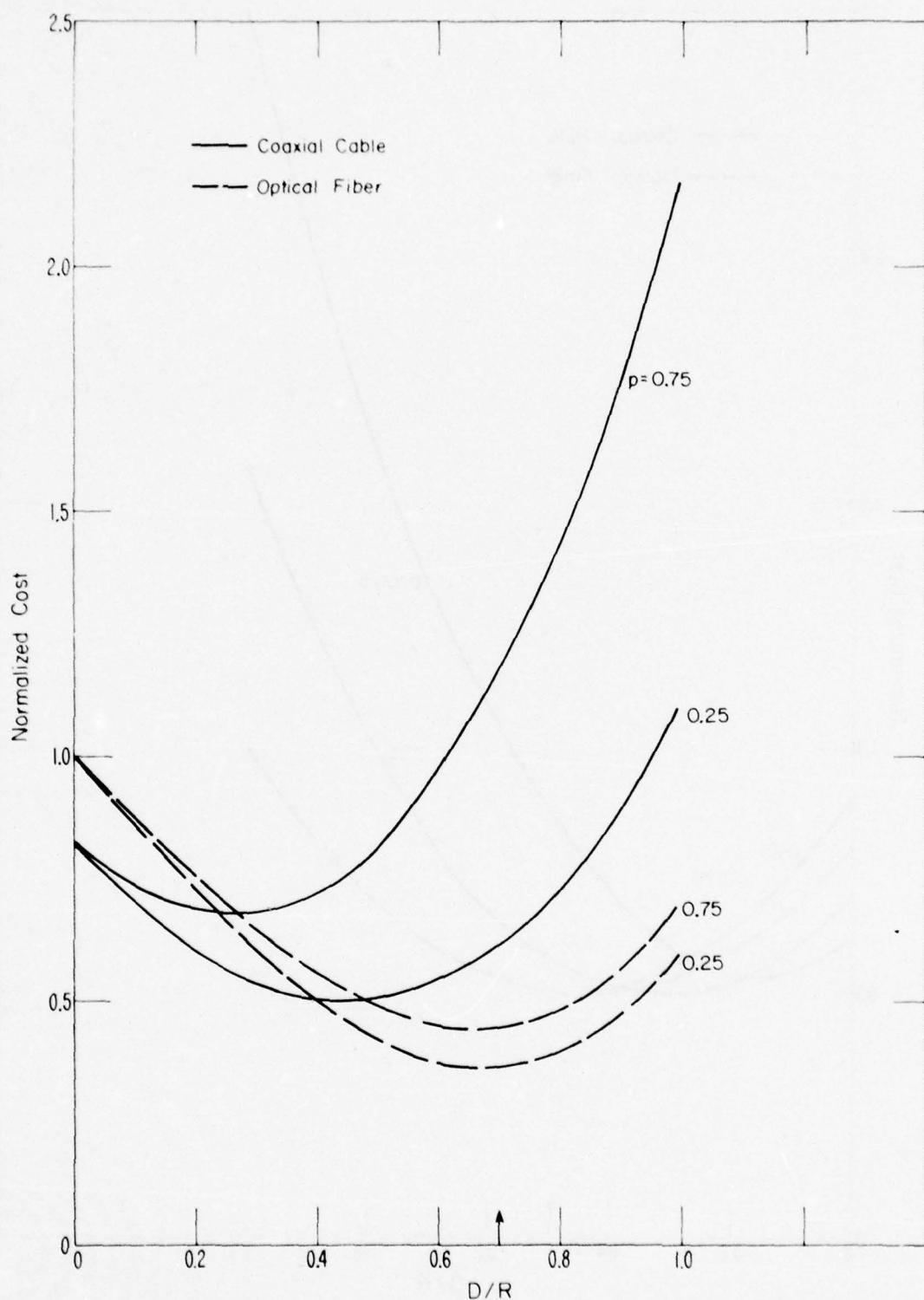


Figure 2-5. Relative cost for fiber and cable systems for Scenario 1 with terminal distribution uniform in the rectangular coordinate system. The arrow on the abscissa marks the value of  $D/R$  for which  $E_x$  is minimum.

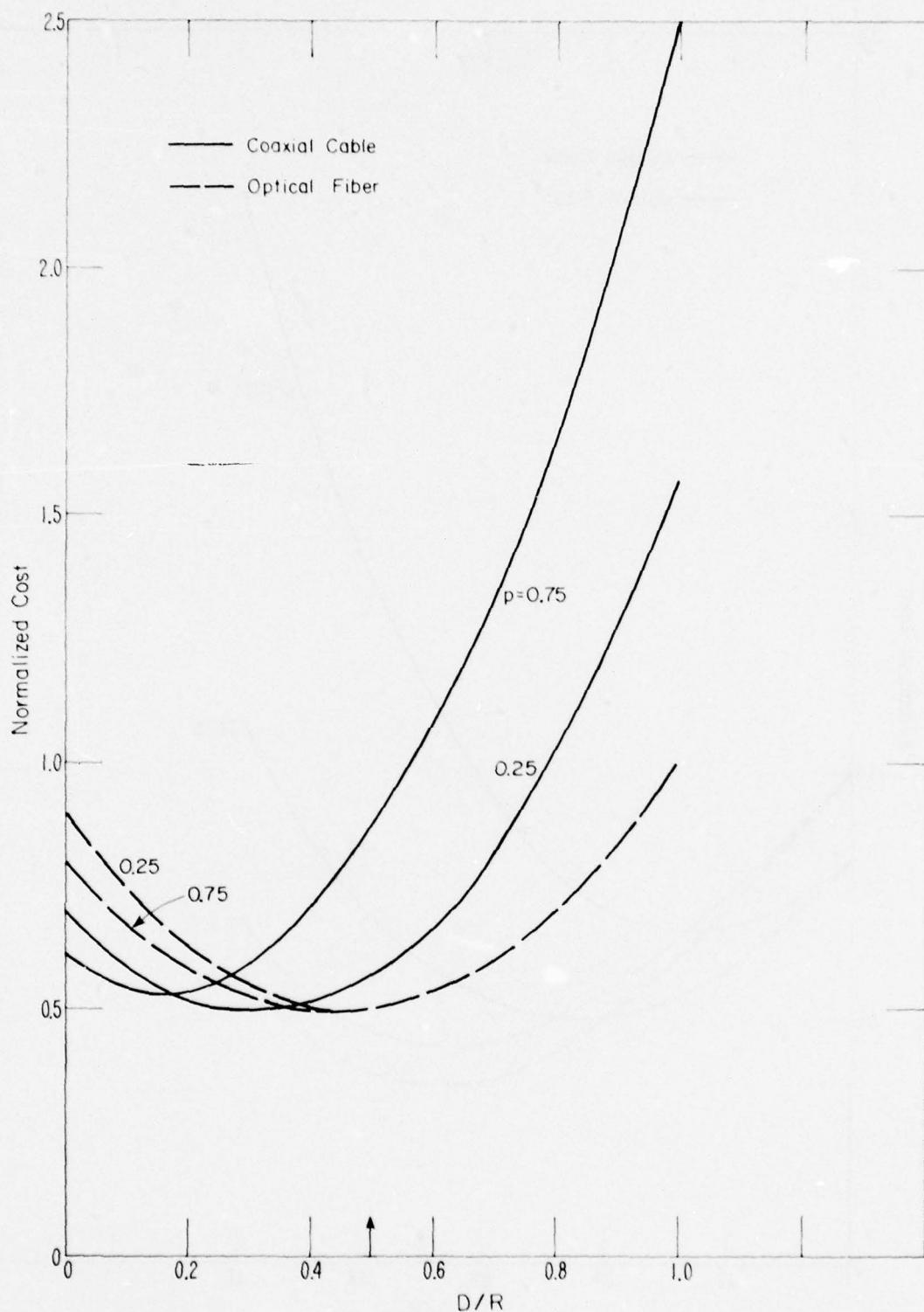


Figure 2-6. Relative cost for fiber and cable systems for Scenario 1 with terminal distribution uniform in the polar coordinate system. The arrow on the abscissa marks the value of  $D/R$  for which  $E_x$  is minimum.

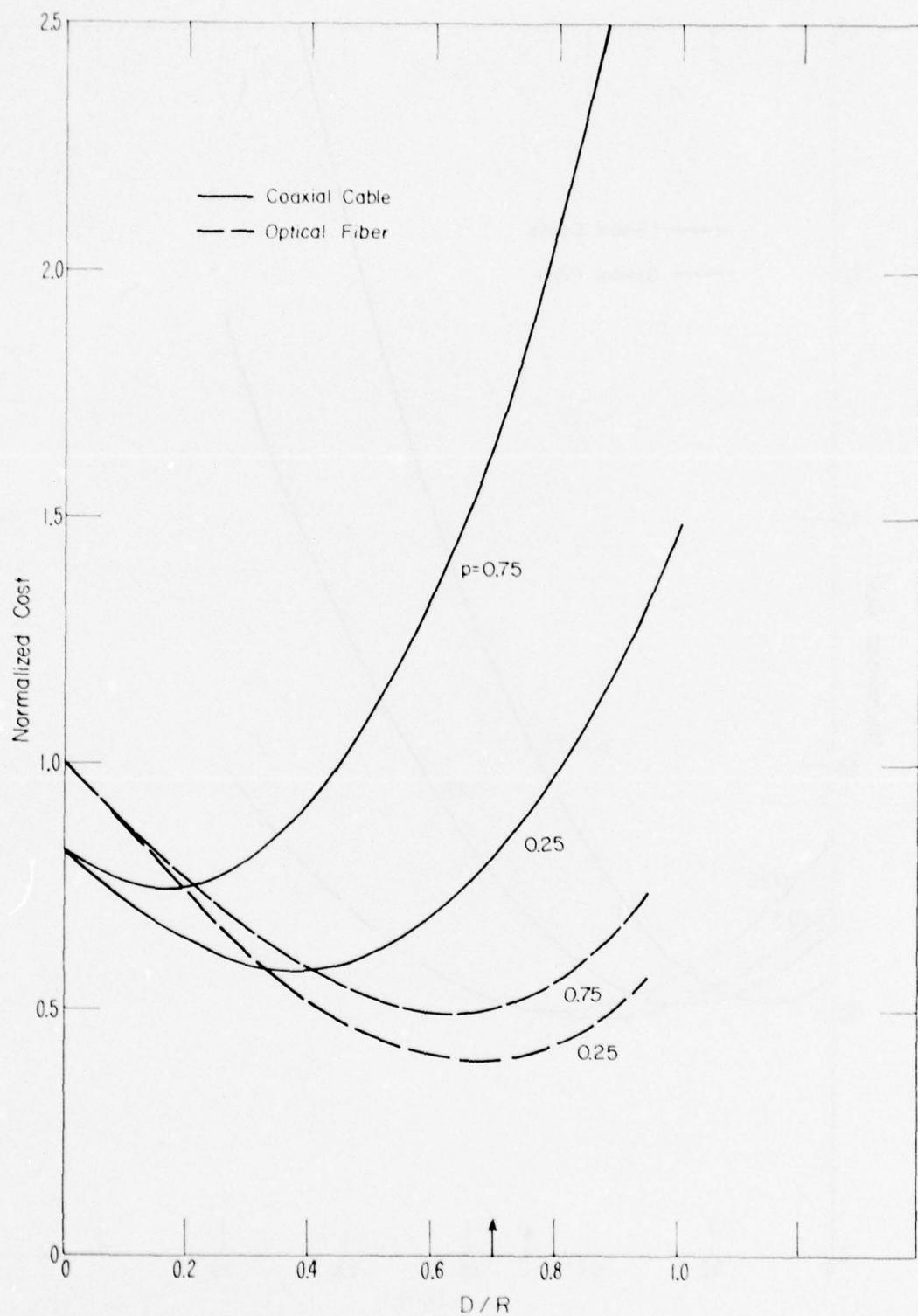


Figure 2-7. Relative cost for fiber and cable systems for Scenario 2 with terminal distribution uniform in the rectangular coordinate system. The arrow on the abscissa marks the value of  $D/R$  for which  $E_x$  is minimum.

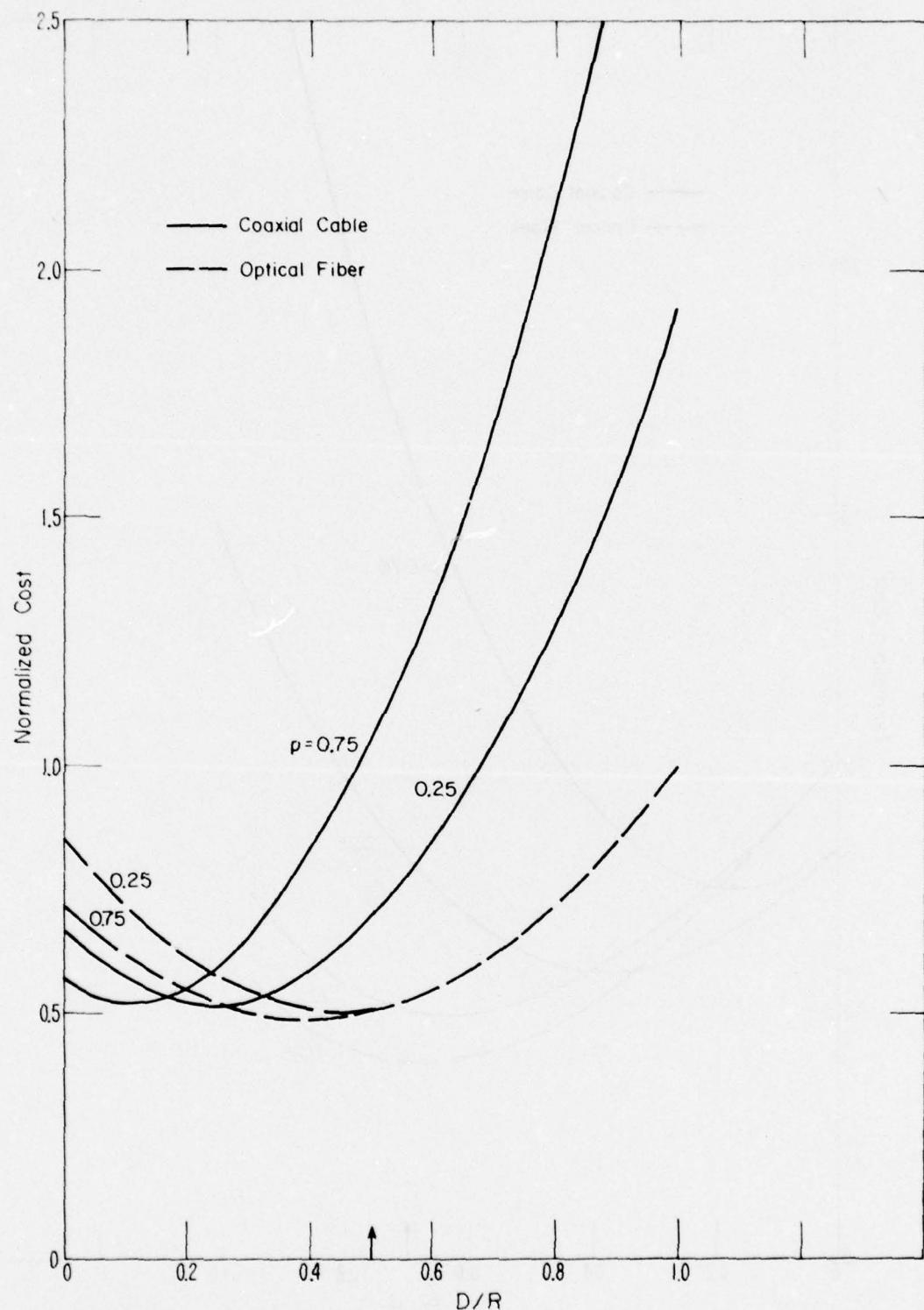


Figure 2-8. Relative cost for fiber and cable systems for Scenario 2 with terminal distribution uniform in the polar coordinate system. The arrow on the abscissa marks the value of  $D/R$  for which  $E_x$  is minimum.

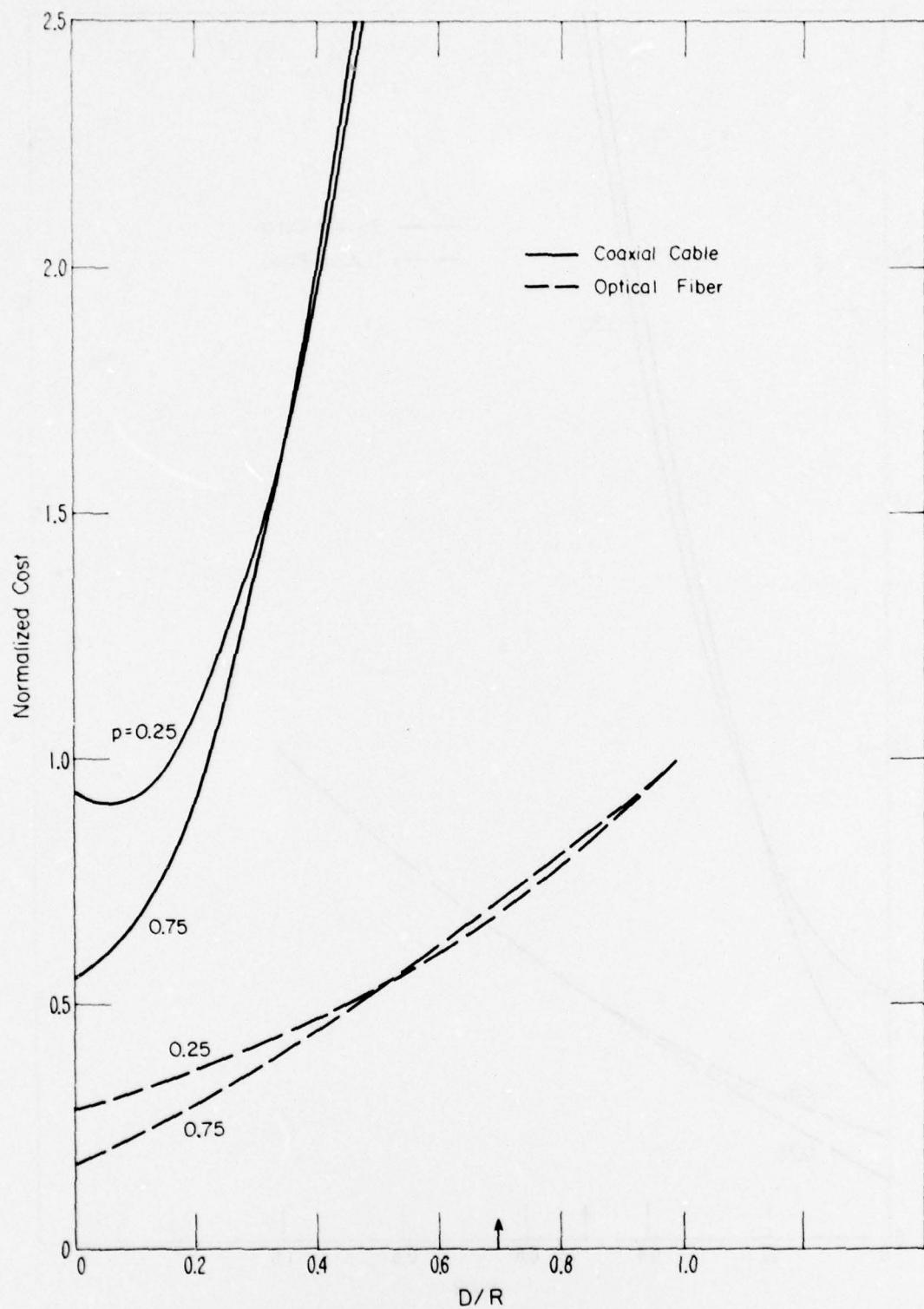


Figure 2-9. Relative cost for fiber and cable systems for Scenario 3 with terminal distribution uniform in the rectangular coordinate system. The arrow on the abscissa marks the value of D/R for which  $E_x$  is minimum.

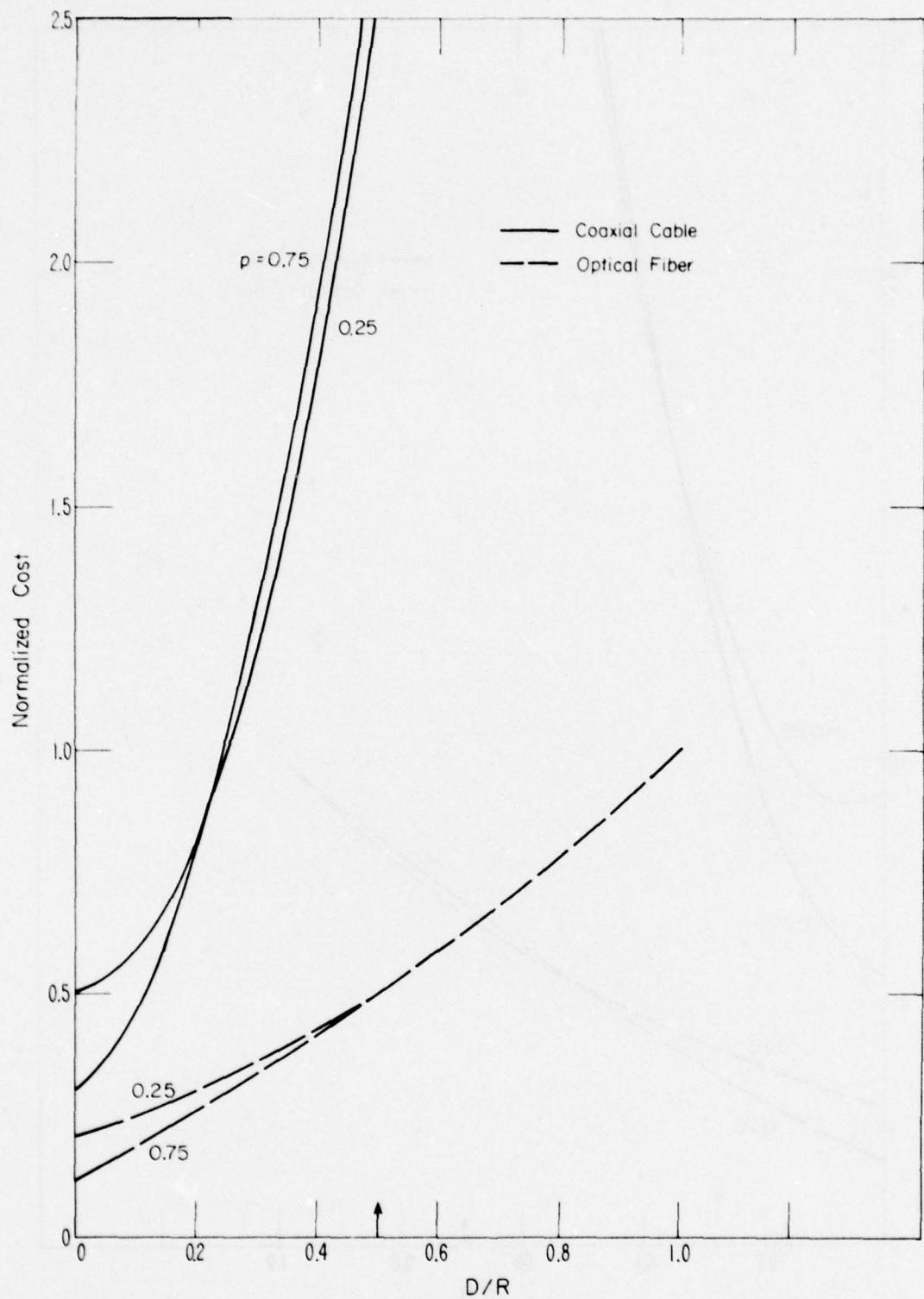


Figure 2-10. Relative cost for fiber and cable systems for Scenario 3 with terminal distribution uniform in the polar coordinate system. The arrow on the abscissa marks the value of  $D/R$  for which  $E_x$  is minimum.

Several points are obvious, as can be seen by examining the figures:

1. For the moderate systems (Scenarios 1 and 2) the minimum cost for the fiber system is nearly at the point for which total line consumption is minimum. This is a bonus since minimum line consumption would minimize degradation and logistics problems. The minimum cost for the cable system is for smaller values of loop radius,  $D$ , and more than the minimum total line consumption.
2. In each case, increasing activity leads to decreasing optimum values of  $D/R$ .
3. The fiber system is invariably more tolerant of mistakes in locating the optimum loop radius; i.e., the minimum for the fiber system is quite broad whereas for the cable, it is relatively sharp. Note, in this regard, that the cable system cost goes up rather abruptly if the loop cannot be placed at the optimum because of obstructions, for example.
4. For the most ambitious system (Scenario 3) the star system ( $D/R = 0$ ) is the most economical in each case except when  $p = 0.25$ , in the cable system. In this system we also see how rapidly the cable system cost increases when  $D/R$  moves away from the optimum.

Table 2-2 will help in the interpretation of the data given in the curves. The 4th and 5th columns give the ratio of fiber cost at its minimum to cable cost at its minimum. Column 4 is for fiber being 50% more expensive than the cable (base cost) and Column 5 corresponds to fiber cost being twice that of cable. Column 6 gives the ratio of base costs required to yield equal system costs at the respective minima. Note that the fiber system is competitive with cable even though its base cost is 1.5 to 2.0 times the base cost of copper. In the most ambitious system (Scenario 3) the ratio of base costs can be as high as 4 to 5 and fiber is still competitive. This is not surprising since the high data rate systems demand components which can easily accommodate the short rise times. The response time of a cable is simply not adequate and this is reflected in the cost, in accordance with the model presented here.

Table 2.2. Cost Comparisons

| Scenario | Distribution of Terminals Uniform In | Activity (p) | Fiber Cost:Cable Cost at Minimum (1) | Fiber Cost:Cable Cost at Minimum (2) | See Note (3) |
|----------|--------------------------------------|--------------|--------------------------------------|--------------------------------------|--------------|
| 1        | x-y                                  | 0.25         | 0.72                                 | 0.98                                 | 2.05         |
|          |                                      | { 0.75       | 0.64                                 | 0.86                                 | 2.34         |
| 1        | z-th                                 | 0.25         | 1.02                                 | 1.36                                 | 1.47         |
|          |                                      | { 0.75       | 0.93                                 | 1.24                                 | 1.61         |
| 2        | (all voice)                          | 0.25         | 0.67                                 | 0.90                                 | 2.24         |
|          |                                      | { 0.75       | 0.66                                 | 0.88                                 | 2.27         |
| 2        | (all voice)                          | 0.25         | 0.96                                 | 1.28                                 | 1.56         |
|          |                                      | { 0.75       | 0.94                                 | 1.25                                 | 1.60         |
| 3        | x-y                                  | 0.25         | 0.31                                 | 0.41                                 | 4.30         |
|          |                                      | { 0.75       | 0.30                                 | 0.40                                 | 5.00         |
| 3        | z-th                                 | 0.25         | 0.39                                 | 0.52                                 | 3.85         |
|          |                                      | { 0.75       | 0.39                                 | 0.52                                 | 3.85         |

(1) Ratio taken when each is minimum. Fiber base cost = 1.5 x cable base cost (as on curves).

(2) Ratio taken when each is minimum. Fiber base cost = 2 x cable base cost.

(3) This is the ratio of fiber base cost to cable base cost required to yield equal cost at the minimum of each.

## 2.6 Concluding Remarks

This section, which concentrates on a comparative cost analysis, reveals the economic advantage of glass fiber waveguides when data rate becomes high. The most ambitious system considered (Scenario 3) involves some high-quality video channels (33 MBPS). The cost comparison shows that the capability of glass fibers leads to significant cost advantages in such high-data-rate systems, according to the model presented.

The model does not consider terminal costs, nor does it attach an economic weighting factor to the many peripheral advantages of the fibers. The terminal costs can be expected to be similar for the two types of transmission lines considered. Conventional (cable) devices are much more refined because they have an established history of performance. Terminal devices suitable for optical fiber systems are not yet established operationally but continued improvement in lifetimes and speed renders such devices extremely suitable for modern communication links. Semiconductor laser diodes (LD's) and light emitting diodes (LED's) are basically very simple devices, capable of direct modulation. Thus, we assume that the terminal costs are similar for the two basic system types considered. Additional discussion on this matter is given by Dworkin, et al. (1975).

Optical fibers have bandwidth advantages, as already discussed, but they also have peripheral advantages which serve further to recommend their use. Even though no economic significance was attached to those advantages, they demand attention when a final decision is made. Some of the more important advantages are the following:

**Crosstalk:** The need for immunity to crosstalk is obvious. The optical fiber enjoys an advantage over conventional transmission lines in this regard. The need for immunity from electromagnetic interference (EMI) is also obvious, and with reasonable care such immunity can be provided with glass fibers.

**Security:** In certain applications, there is a critical need for secure communications. The optical fiber, while not totally secure, provides a vast improvement over conventional lines.

An unauthorized receiver would require elaborate and sophisticated equipment to conduct covert activities.

**Access:** Optical fibers show promise of being amenable to rather simple accessing techniques. The losses can be controlled with a little care in manufacture and, furthermore, flexibility in the approach seems plausible. Optical access couplers will likely be based on simple geometric methods.

**Size and Weight:** The model presented in this section gave some attention to the size of the cables but did not account for basic difference in size for the two types of transmission line. Obviously, increased size and weight will be more difficult and more expensive to install. This was not accounted for in the model of this section.

### 3. THE INTERCONNECTION OF CLUSTERS

#### 3.1 Introduction

The previous section concentrated on a uniform distribution of terminals in a circle and we considered only the star and loop topologies. Indeed, the star is merely a special case of the loop configuration. The analysis, while obviously restricted, gives insight into the cost penalty of placing the loop incorrectly. The model also gives an economic basis for decision making when the terminals are densely populated. In some other cases, the distribution of the terminals is clustered, such that an isolated part of the network involves several terminals (a cluster) which may interconnect as a star or loop network. The cluster is a local subnetwork which must be connected economically to a similar subnetwork clustered at another location in the overall network. Each subnetwork may involve many terminals, interconnected in a fashion described in the previous section. The concept is illustrated in Figure 3-1. In the final analysis, an ARBITS network will involve the interconnection of a wide variety of configurations.

When connecting many subnetworks, the connecting line can involve a considerable expenditure and one is led to an interesting problem concerned with a minimum consumption of line interconnecting those clusters. This section is concerned with the minimization of line consumption in an interconnecting network.

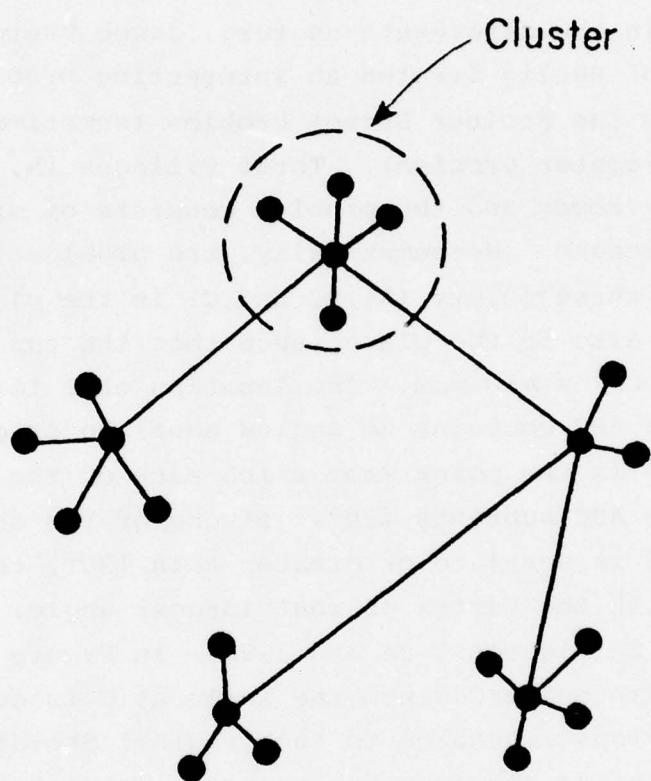


Figure 3-1. A cluster of terminals.

In what follows, we will use the term "node" to refer to the clustered subnetwork. Concentration will be on the connection of those nodes.

### 3.2 The Steiner Street Problem

Early in the nineteenth century, Jacob Steiner of the University of Berlin treated an interesting problem in geometry, now known as the Steiner Street Problem (sometimes referred to as the traffic-center problem). Three villages (A, B, and C) are to be joined by roads and the problem consists of minimizing the total road length. Mathematically, the problem is attacked by considering three points (A, B, and C) in the plane and seeking a point, P, also in the plane, such that the sum of the distances AP, BP, and CP is minimum. The location of P is as follows: If the triangle ABC contains no angles equal to or greater than  $120^\circ$ , then P is the point from which each of the three sides of the triangle ABC subtends  $120^\circ$ . If one of the angles in the triangle ABC is equal to or greater than  $120^\circ$ , the point P coincides with the vertex of that largest angle. These facts are illustrated in Figures 3-2a and 3-2b. In Figure 3-2b, point P coincides with point C since the angle at C is at least  $120^\circ$ .

The obvious extension to the original Steiner problem involves N points and asks for a single point P in the plane such that the sum of the interconnecting distances is again minimum. That problem is the one encountered in locating the star coupler. The extension of interest here allows more than a single connecting point. Concentration is then on a street network for which the total length is minimum. The problem is to find a system of connected straight-line segments of minimum total length such that any two of the N points can be joined by segments of the system. It is this problem which is of interest in the interconnection of the clusters, as in Figure 3-1. In that case, the segments are not streets and the points are not villages; instead, concern is with a minimum length of transmission line connecting the terminals. The resulting network will be referred to as a Steiner minimal tree.

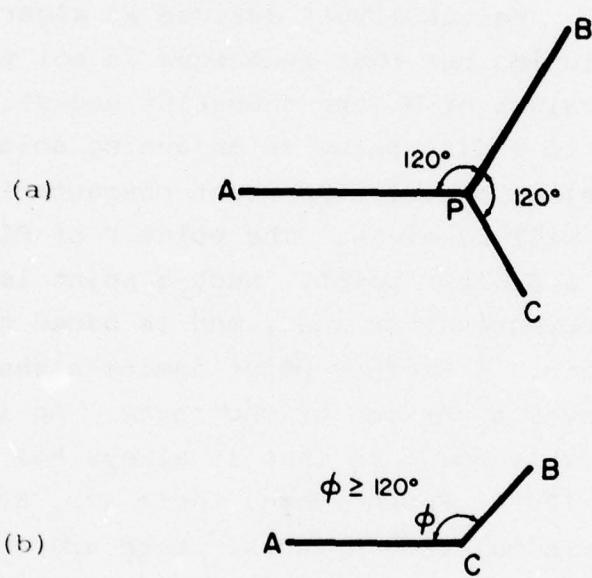


Figure 3-2. Steiner points under two topographical situations.

The solution to the Steiner Street Problem to yield a Steiner minimal tree has received attention with regard to cross-country telephony routes where the cost of right-of-way and installation has significant economic impact.

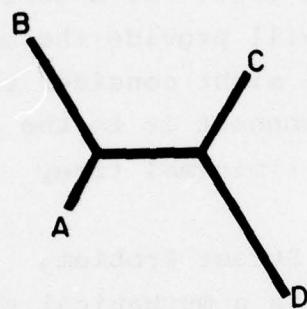
Gilbert (1967) and Gilbert and Pollack (1968) discussed a geometric method of finding the Steiner minimal tree but, in practice, such a method would have minor disadvantages, which will be discussed below. Melzak (1961) derived an algorithm for finding the desired solution but that technique is not practical except for very small values of  $M$  (the number of nodes).

Before proceeding to a discussion of an analog solution to the Steiner Street Problem, several important characteristics of a Steiner minimal tree will be given. The point  $P$  of Figure 3-2a will be referred to as a Steiner point. Such a point is a vertex, not coinciding with a communication node, and is added to allow a reduced total line length. A Steiner point admits a sharing of traffic between nodes over a portion of the route. An important characteristic of a Steiner point is that it always has exactly three lines meeting at  $120^\circ$ . Furthermore, there are, at most,  $M-2$  Steiner points in a minimal tree network; there are exactly  $M-2$  such points only if every node connects to only one line;  $M$  is the number of nodes to be connected.

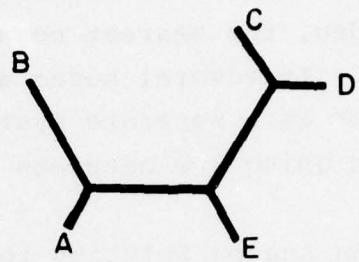
A Steiner tree may not be a Steiner minimal tree. A relatively minimal tree has been found if a small perturbation in the topology does not shorten the tree. Every topology admits at least one relatively minimal tree; that tree may, of course, also be the Steiner minimal tree that is being sought.

A Steiner tree contains no pairs of lines which meet at less than  $120^\circ$ . For communication nodes which lie such that direct connections would violate this rule, the introduction of a Steiner point will yield reduced length and compliance with the rule on angles.

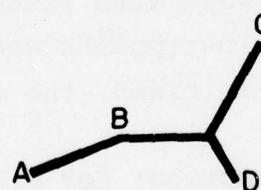
Figures 3-3a and 3-3b show arrangements for  $M=4$ , 5, for which there are  $M-2$  Steiner points. Figure 3-3c shows a configuration ( $M=4$ ) for which there is only one Steiner point. The



(a)



(b)



(c)

Figure 3-3. Four and five terminal configurations.

original Steiner problem ( $M=3$ ) is important when nodes are in clusters as shown in Figures 3-3a and 3-3b. If a single node (or a cluster of nodes) is relatively isolated from the centroid of all nodes, the advantage of the Steiner tree is diminished.

When new nodes are added to an existing Steiner tree, connections are based on the concepts of the Steiner minimal tree and connections to the nearest existing line. If a single node is added, the nearest connecting line will provide the connecting point. If several nodes are added, one might consider the new cluster as a separate system and then connect it to the existing system using the concepts of the Steiner minimal tree.

### 3.3 An Analog Solution to the Steiner Street Problem

A Steiner tree can be interpreted as a mechanical system for which the potential energy is related to the distance between nodes. Minimum total line length is then associated with minimum potential energy, which is naturally sought by the system if equilibrium is to prevail. One such mechanical system can be had by using elastic bands, always with fixed tension, stretched between nodes. If the Steiner points are free to move and the communication nodes are held fixed, the equilibrium position is a relatively minimal tree.

This analog is not practical for  $M>3$ , so we turn to another method. A technique can be envisioned by considering a soap film. The surface tension of the film is capable of doing work and the energy expended by the film will be directed to a minimization of potential energy within the constraints of the system. In the case of a soap bubble, the film surface tension forces the surface area enclosing the specified volume of air to be minimum; the result is a spherical surface.

Soap films can be used to solve the Steiner street problem by relying on the surface tension of the film to minimize the area of a suitably restricted surface (Courant and Robbins, 1969). Two parallel smooth plates of glass or plastic are joined by  $M$  perpendicular bars. The bars are located according to an

appropriate scale for the Steiner street problem under consideration. If the object is immersed in a soap solution and withdrawn carefully, the film forms a system of surfaces between the two plates and joining the fixed bars. The intersection of the film surfaces and the glass plates represents the solution to the Steiner Street Problem. The expected result for a configuration for which  $M=5$  is shown in Figure 3-4. Photographs of soap films produced in the laboratory are shown in Figures 3-5 to 3-7. The interested reader will find a wealth of descriptive material and color photographs in the references (Almgren and Taylor, 1976; Isenberg, 1976) which deal with the geometry of soap films.

Figures 3-5 and 3-6 are, respectively, an angle view and a top view of the same four-station model. Figure 3-5 gives a perspective of the film while Figure 3-6 is appropriate for use in actually scaling the layout to determine node location. Figure 3-7 is a top view of the film produced in a model simulating six interconnecting nodes. Note that there are  $M-2$  Steiner points in these figures. The configuration of Figure 3-7 represents about 22% improvement over the star counterpart in the expenditure of transmission line.

### 3.4 The Steiner Tree System

There is an immediate concern in applying the Steiner Street solution to an interconnection problem. It is seldom possible to place connecting links according to an arbitrary geometry. There are usually physical obstructions which preclude a link's traversing a desired path. However, this impediment would also hinder the layout and operation of other networks. Indeed, this impediment makes it even more important that the design engineer know the connecting geometry which minimizes total path length.

As the communicator experiments with the concepts discussed above, he will discover that the relatively minimal tree may emerge for his particular street problem. Upon examination, he may discover that he has several options in route definition.

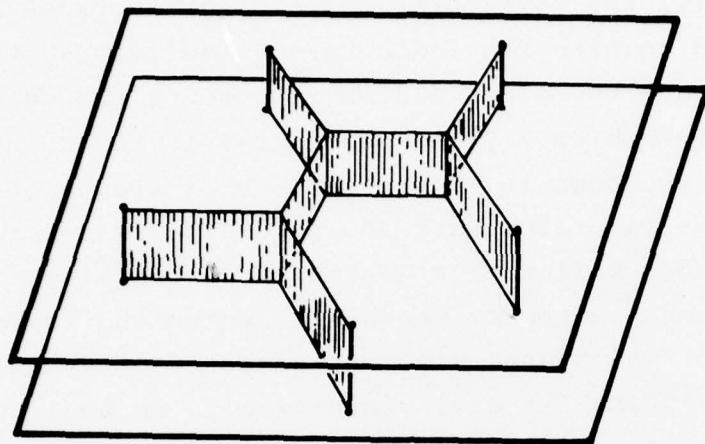


Figure 3-4. Expected soap film for  $M=5$ .

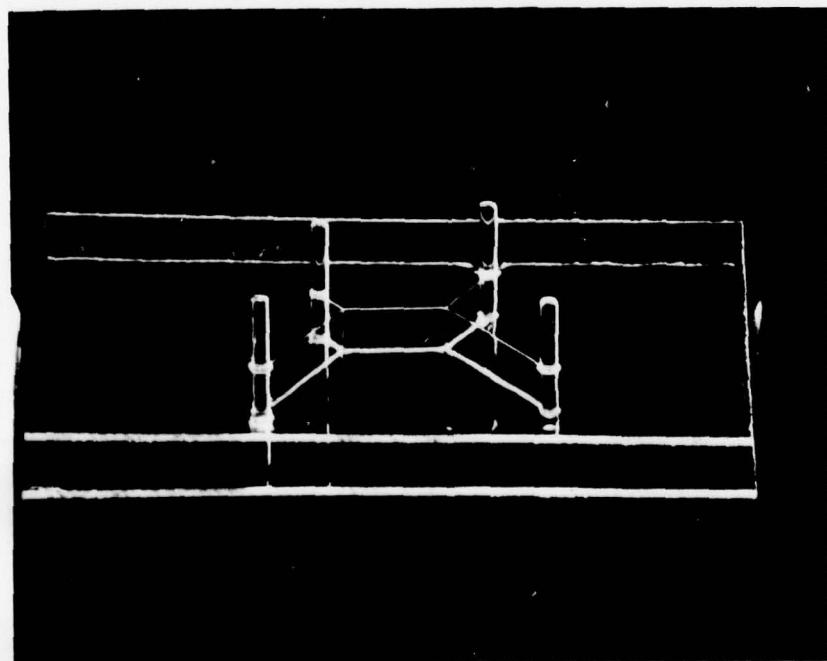


Figure 3-5. Side view of photograph of soap film produced in the laboratory to illustrate the technique.  $M=4$ .

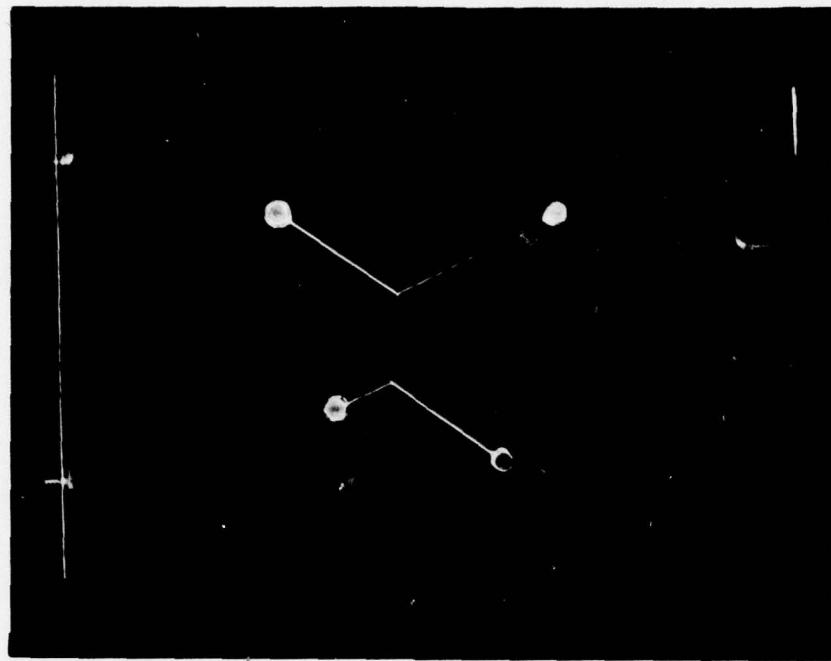


Figure 3-6. Top view of film of Figure 3-5.

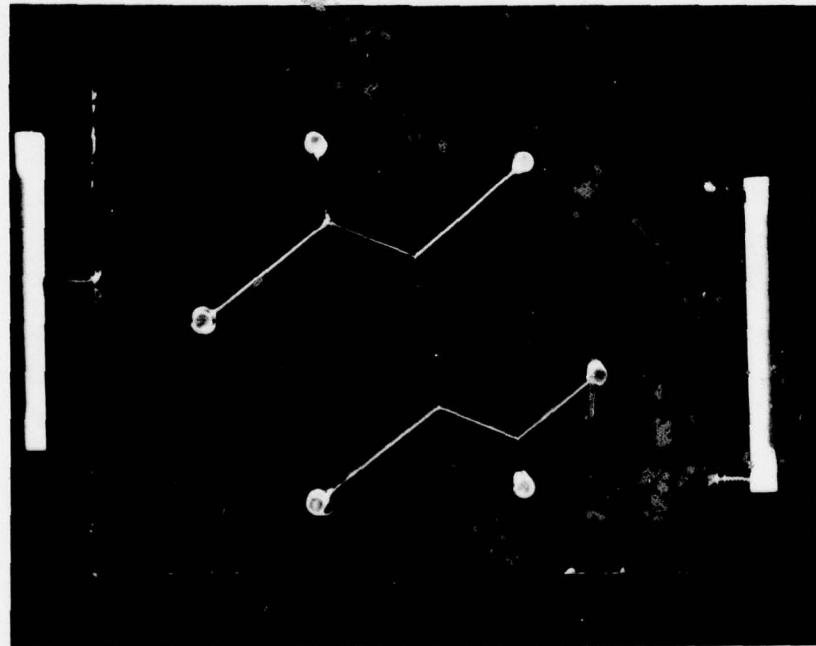


Figure 3-7. Top view of film when  $M=6$ .

The difference in total line length for the various options is usually quite small. The soap films will be stable in the relatively minimal configurations. Experimenting with the films will be informative and the route alternatives which emerge will provide opportunities to avoid physical obstructions on the actual path.

A case in point is illustrated in Figures 3-8 and 3-9, which show two films produced with the same 5-terminal model. Both films are in stable equilibrium and have formed because of the way the models were withdrawn from the bath. The total path length is approximately the same (within about 4%) in the two cases and each is less than the total path length for the star configuration by about 20%.

Figures 3-8 and 3-9 illustrate a bonus available from the use of this analog solution. If terrain precludes locating a node in a particular area or stringing the transmission line along a particular path, alternatives may have to be examined. Figures 3-8 and 3-9 show two such alternatives for the five-terminal network. In experimenting with a model, one may find several such solutions, each of which represents a possible interconnect geometry. By scaling and measuring, we find only a small difference in total path length between the alternatives and each represents a significant reduction in total path length from that found in the star system.

If an area is to be precluded, this can be accounted for as shown in Figure 3-10, which shows the precluded area as a circle. In the model, a circular hole is drilled to scale in the parallel plates to represent the precluded area. In the figure, 3 nodes are shown. The soap film can be made to avoid the area by breaking any film that forms in the hole. Notice in the figure that the lines, representing the soap films, will avoid the holes. An area of any shape can be handled in this way.

The experimenter should be aware that angles of  $120^\circ$  or more are special cases leading to in-line configurations. Blowing on the film causes it to move between the plates. A test of

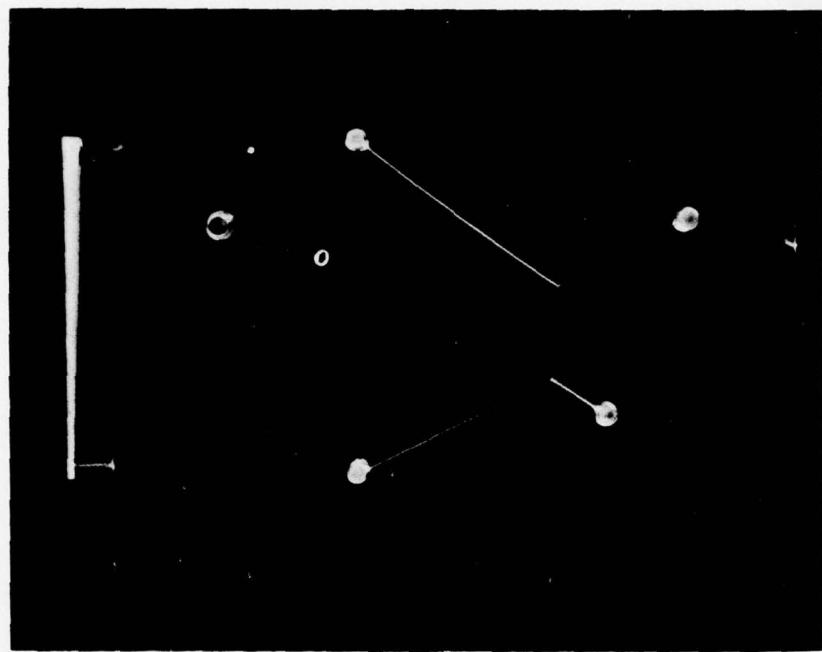


Figure 3-8. Top view of film when  $M=5$ .

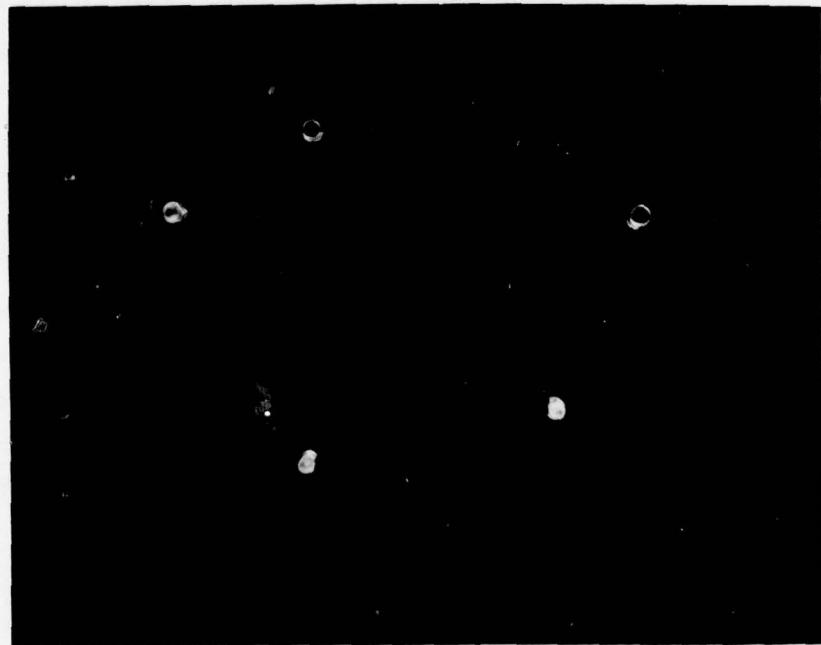


Figure 3-9. Alternate film when  $M=5$ .

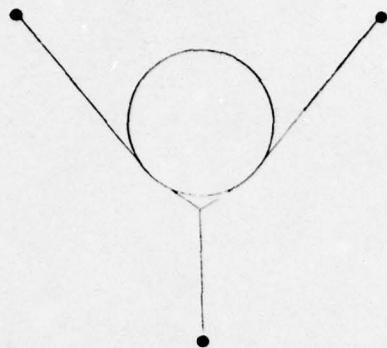


Figure 3-10. Showing how the model can preclude an area from consideration.

minimum distance can be designed around such film movements. A little experience will yield insight in this regard, and the communicator with a complex data-bus topography should consider an in-depth study of the options presented.

The use of glycerin in the mixture allows easy movement of the film. Best results are obtained if the model is as small as is practical. The two parallel plates of the model should be as close together as is practical.

### 3.5 Comments and Comparison with the Star System

In the connection of the nodes, one with the other, we could resort to the star configuration, if the distribution of the clusters is suitable. Indeed, the interconnection of the clustered subnetworks represents a topological problem quite like the corresponding problem of interconnecting the communication terminals within the subnetwork. It is worthwhile to compare the distances involved in the Steiner geometry with corresponding distances in alternative geometries. The star system is a special case which is amenable to easy comparison and so it will be used in these comparisons. In addition, we offer here some comments which have bearing on the selection of an interconnection geometry.

1. Referring to Figure 3-2b, note that the maximum length of line required to connect two points via a Steiner point is  $1.15d$  where  $d$  is the straight-line distance between the two points. The angle  $\theta$  of the figure must be at least  $120^\circ$ , in order for point C to be a Steiner point. The maximum line length occurs when points A and B are equidistant from C and  $\theta=120^\circ$ . In that case, the distance from A to B, via C is  $1.15d$ .
2. If the nodes in question are distributed according to the suggestions of the previous section (see Figures 2-2a and 2-3a and the corresponding discussion in Section 2.2), then the expected distance between nodes is as shown in Figures 2-2b and 2-3b. For the distribution which is uniform in rectangular coordinates (Figure 2-2a), the expected distance is about  $0.85R$  where  $R$  is the radius of the service area (see Figure 2-2b). For the  $r-\theta$  uniform distribution, the expected distance is about  $0.6R$  (see Figure 2-3b).
3. The average distance between two points in the star configuration is  $\frac{4}{3}R$  for the distribution which is uniform in the x-y coordinate system. The corresponding distance for the  $r-\theta$  distribution is  $R$ .

### 3.6 Concluding Remarks

In the interconnection of isolated clusters of communication nodes, line consumption will be an important consideration. The concepts discussed in this section are intended to point out the advantage of locating Steiner points in the system to optimize the total line length. For the cases we considered, the Steiner minimal tree used about 20% less line than the star system in a typical geometry. This could be predicted by beginning with a simple four-terminal network wherein each terminal is at the vertex of a square. Comparing the two, we find the Steiner tree provides savings in transmission line length of about 20%. Building on that simple four-terminal system, we speculate that 20% savings in line length is not unusual. In practice, of

course, a model could be constructed to scale and the savings in line length can be measured. There will be special cases for which there will be no savings in transmission line.

#### 4. OPTIONS IN OPTICAL COMPONENTS

##### 4.1 Introduction

In the preceding sections, we gave serious consideration to the economics and feasibility of using optical waveguides and the associated optical components to meet the demands of ARBITS. Not specifically addressed in those sections, is the practical aspects of accomplishing the access to the optical waveguide. In coaxial cable systems, options are rather limited and the techniques are well understood, since the cables have been in use for some time. In the optical regime, several reasonable options exist, each with its advantages and disadvantages. In this section, we will discuss some of those options, giving special attention to couplers, since this is a key component when the number of terminals becomes reasonably large.

There are two distinct approaches to the use of optical waveguides in a multiterminal network such as ARBITS: (1) the use of fiber bundles with appropriate multiplexing schemes and, (2) the use of single fiber waveguides in either a single fiber cable or a multi-fiber cable with each fiber acting as an independent channel. In the second case, several multiplexing schemes are available aside from the obvious use of space division multiplexing (separate channels on separate fibers).

A typical geometry for a fiber bundle is shown in Figure 4-1. Variations in mechanical designs for a cable containing 6 or 7 individual fibers are shown in Figure 4-2. Figure 4-3 shows a cable which incorporates 6 or 7 cables of the type shown in Figure 4-2, in each of 6 or 7 cables, all contained within a larger cable containing a total of 6x6x6(or 7x7x7) fibers.

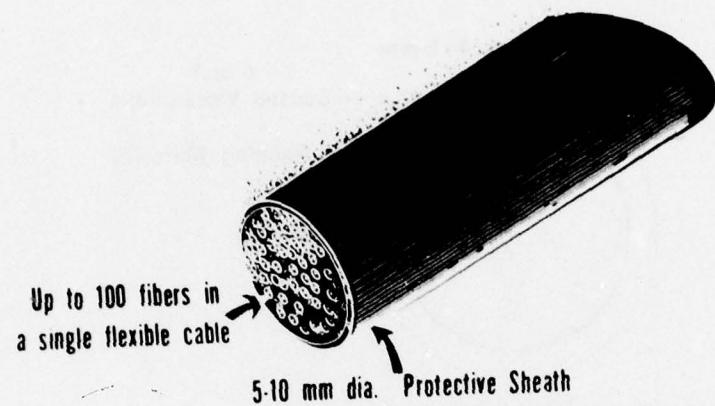


Figure 4-1. Fiber bundle.

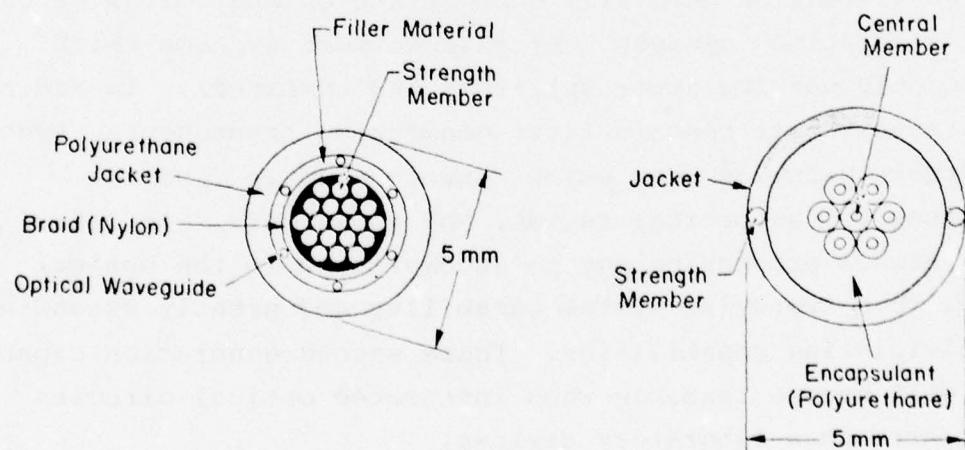


Figure 4-2. Variations in cable configuration.

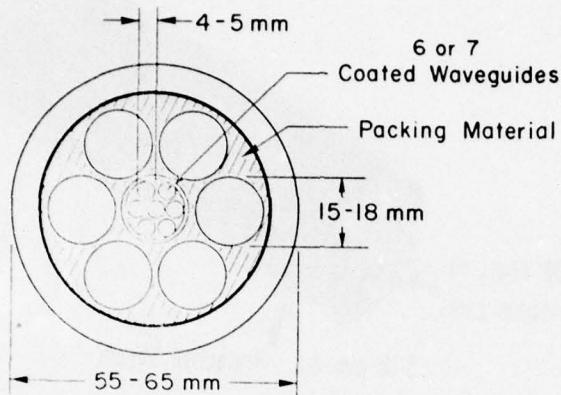


Figure 4-3. Possible cable configuration containing 6x6x6 or 7x7x7.

The discussion here will concentrate on what might be called "first generation" systems. By this we mean systems which contain only passive power splitters and combiners. In addition, the terminals will contain first generation transducers, such transducers being devices which convert signals from the electrical to the optical regime, and conversely. In later years, signal processing may be accomplished in the optical regime, thus improving system capability and greatly extending the multiplexing capabilities. These second-generation capabilities will become feasible when integrated optical circuits become more than laboratory devices.

#### 4.2 Fiber Bundles

In certain restricted network geometries, it may be feasible and attractive to use fiber bundles in the distribution system.

It represents an impractical approach when the physical size of the network exceeds a few hundred meters in diameter since the cost of fibers tends to be on the basis of fiber-meters used. Thus, if each bundle contains 100 fibers (say), then the cost will be greater, generally than for a cable containing only a few fibers. This is a slight oversimplification since the fibers generally used in bundles are of moderate quality since the distances usually involved are only moderate. Conversely, cables containing only a few fibers (Figure 4-2) tend to use high quality fibers, which are more expensive. It is conceivable that a given ARBITS geometry might contain several intrabuilding terminals or a cluster of terminals in adjacent buildings, for which the use of bundles would be most appropriate. In this case, there might be a prolific use of single-fiber cable outside the confines of such short-distance interstructure and intra-structure links. The terminals inside a command post, for example, could be interconnected via fiber bundles with a suitable interface forming the connection with terminals and single fiber cable outside such a command post.

Fiber bundles offer the distinct advantage of redundancy. All the fibers (see Figure 4-1) are illuminated simultaneously and all carry the same information. If some of the fibers are broken in the process of installation, it is not of serious consequence. By the same token, connection is rather simple and the bundle cross section is sufficient to allow reasonable area match to most sources and detectors.

One of the key disadvantages of fiber bundles is the packing fraction loss at each connection and at sources. Packing fraction loss comes about because only the core area of a fiber is useful in carrying information but the end of the fiber bundle exposes not just core area but also cladding and space between fibers. The ratio of total core surface area to the area of the bundle is called the packing fraction (PF). In a hexagonal close pack, shown in Figure 4-4, packing fraction is maximized. The figure shows the geometric construction which can be used to show that packing fraction can be calculated from Equation (4-1):

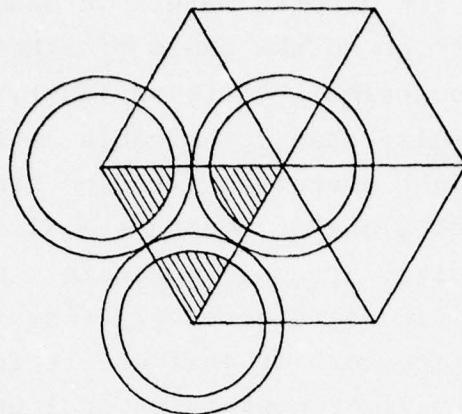


Figure 4-4. Geometry illustrating hexagonal close pack.

$$PF = \frac{\pi}{2\sqrt{3}} \left( \frac{\text{Core diameter}}{\text{Cladding diameter}} \right)^2. \quad (4-1)$$

When the term in parentheses is unity (no clad), PF is maximum, the only loss being due to the space between fibers. In that case,  $PF = 0.91$ . Note that loss due to Fresnel reflection at the interface is distinct from and not included in packing-fraction loss. Fresnel reflection loss is encountered each time a ray bundle encounters a change in refractive index. Thus, at the end of the fiber bundle, part of the signal energy is reflected by the fiber-air interface. Similar loss is encountered when energy first enters the fiber. For a typical refractive index of glass ( $n = 1.5$ ), the reflection coefficient for normal incidence on a plane polished fiber end is 0.04; thus, 92% of the incident light is transmitted into the fiber core.

Unfortunately, packing fraction loss is suffered often in a multiterminal network. This will become clear in what follows.

#### 4.2.1 Star and tee systems

In the two preceding sections (2 and 3), we discussed alternatives in power splitters and combiners for multiterminal networks. It is obvious from that discussion that one needs to

consider two basic forms of such splitters and combiners: (1) an in-line or tee configuration and (2) a star configuration. The tee configuration will be used any time three lines are interconnected. This will be the case in the tee and the loop configurations. It will also be the case for Steiner points (see Section 3). In the latter case, it will often be convenient to seek a 3 dB coupler. In that case, the configuration shown in Figure 4-5 may be useful. Variations of that theme could also be used to vary the degree of power split. A refinement of the method illustrated in Figure 4-5 would use a locally increased refractive index in the low-loss glass to direct the signal power from Port 1 (say) to Ports 2 and 3 by virtue of the increased

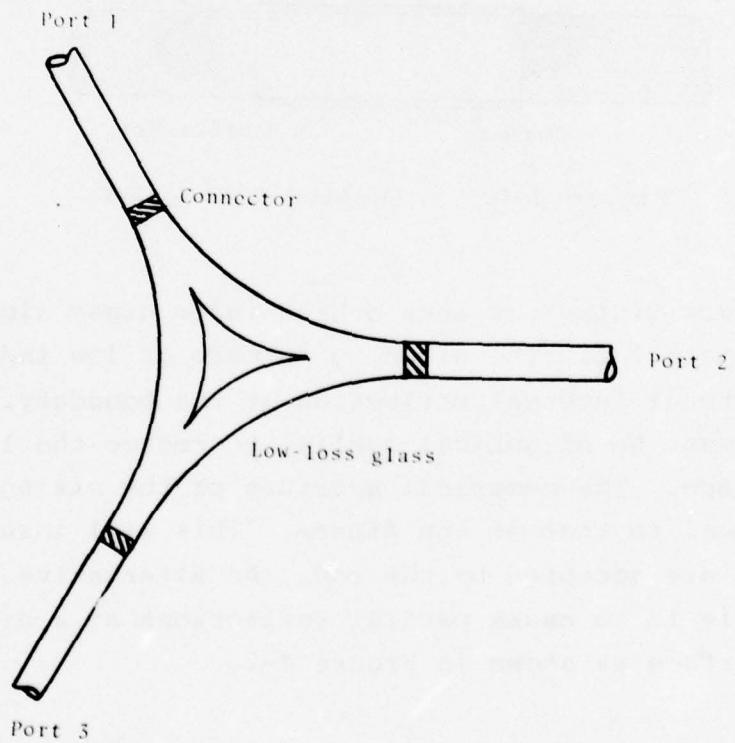


Figure 4-5. Possible 3 dB splitter.

refractive index which tends to channel signal energy in a manner analogous to the physical constraint shown in Figure 4-5.

Another possible access for the in-line system is via a furcation of the fiber bundle and a mixer or scrambler rod (see fig. 4-8). A mixer (coupler) is shown in Figure 4-6. In the furcation scheme, the fiber bundle is on each end of a glass rod. At the input to the rod, each fiber in the bundle illuminates the rod; the length of the rod is sufficient to allow mixing of those inputs; the exit of the rod is thus uniformly illuminated so each fiber at the exit face carries the same signal. The function of the mixing rod is thus to integrate or scramble the incoming signal over the entire exit aperture. The total signal is then available at every station.

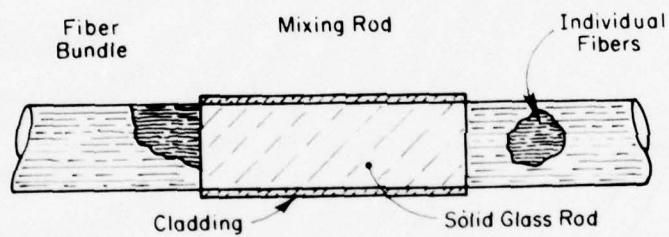


Figure 4-6. In-line mixing rod.

The glass mixing rod uses a high index glass similar to the fiber core material. The cladding is made of low index material to provide total internal reflection at the boundary. The ends of the rod must be of optical quality to reduce the losses due to that interface. The numerical aperture of the mixing rod must be at least equal to that of the fibers. This will insure that all of the rays are accepted by the rod. An alternative to furcation of the bundle is to cause partial reflections at a dielectric or metallic surface as shown in Figure 4-7.

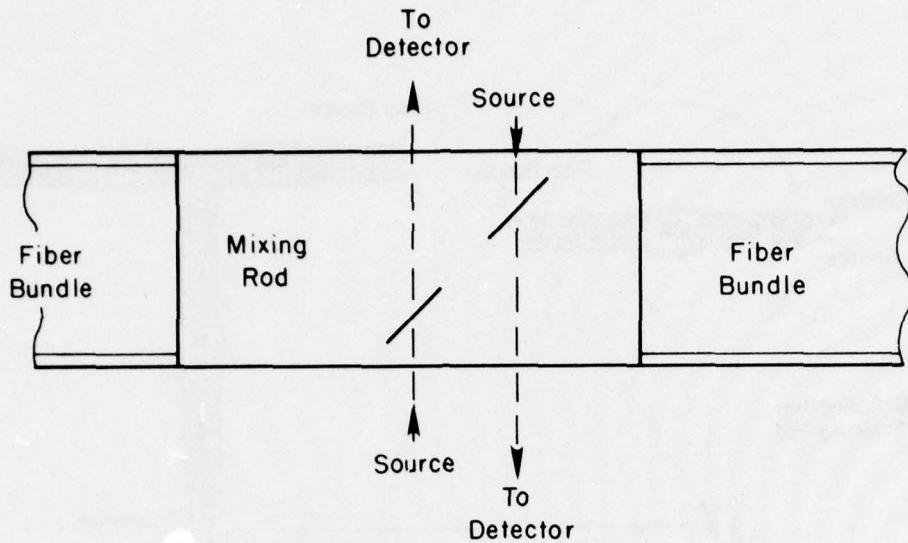


Figure 4-7. Access coupler.

The losses include the usual connector loss and the packing fraction loss.

The geometry of an  $N$ -station tee system is shown in Figure 4-8. Since, in the extreme case, the signal must pass through all the couplers, if each coupler introduces a fixed loss (in dB) then the total loss of the system (worst case) is the sum of the losses; total loss thus increases linearly with  $N$ .

Consider  $N$  terminals, as shown in Figure 4-8. To illustrate basic concepts, we ignore the fiber loss and concentrate on the losses introduced by the coupling hardware. Designate the coupling loss, in dB, as  $L_d$ , where the subscript  $d$  alludes to the fact that a fraction of the power is divided at the tee coupler, allowing a fraction  $(P_d/P_i)$  of the incident power to be split off. In some cases,  $P_d/P_i$  is taken as approximately 10%. A fixed fraction of coupled power is more desirable since that would allow use of a standard component (coupler), independent of the number of stations.

$$L_d = 10 \log (P_d/P_i). \quad (4-2)$$

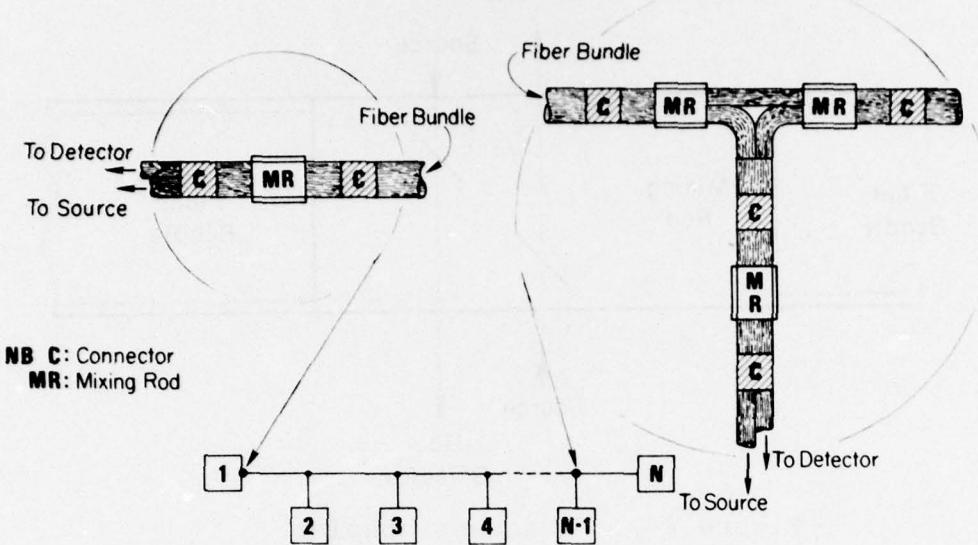


Figure 4-8. Tee system configuration.

The fraction of incident power transmitted through the coupler (subscript t) is

$$\frac{P_t}{P_i} = \frac{P_i - P_d}{P_i} = 1 - \frac{P_d}{P_i}.$$

The insertion loss is

$$L_{it} = 10 \log (1 - (P_d/P_i)). \quad (4-3)$$

If  $L_d = -10$  dB,  $L_{it} = -0.46$  dB.

Packing-fraction loss at each coupler depends on the geometry of the fibers; this was discussed earlier. For this discussion, designate that loss as  $L_{PF}$ . This term can include surface reflection losses, but PF is the dominant factor.

$$L_{PF} = 10 \log (PF). \quad (4-4)$$

There is another furcation at each terminal since it contains both a detector and a source. The associated loss,  $L_f$ , is usually -3 dB. This loss must be suffered to provide full duplex service. Finally, the loss suffered in each connector is designated  $L_c$ .

The total loss between the output of terminal 1 and the input to terminal  $N-1$ , which is usually the worst case situation, is given as follows:

$$L_{wct} = 5L_c + 3L_{PF} + L_f + (N-3)(2L_c + 2L_{PF} + L_{it}) \quad (4-5)$$

In practice, it may be possible to eliminate some of the connectors, depending on their structure and the nature of the mixing rod.

The loss, in dB, increases linearly with  $N$  in the tee distribution system. We will find that the star system has a distinct advantage, in this regard. However, the tee system has an advantage in surviving outage because of damage: if the main trunk breaks, the system will become two independent and unconnected trunks. Damage to a single terminal will not affect another terminal. There is no isolated outage which will render the entire system totally inoperative. This is not the case in the star distribution system.

A possible mixing rod for the star system (Figure 4-9) is similar to the tee mixing rod, but has one mirrored end (Hudson & Thiel, 1974; Milton & Lee, 1976). The mirror causes each incoming signal to be uniformly distributed among all fibers. The mixing is accomplished, as in the rod of Figure 4-6, through the reflections off the rod wall. If the rod is long enough, the signals will be thoroughly mixed. The mixing rod serves the purpose of connecting each terminal to all other terminals. The resulting distribution network is shown in Figure 4-10. Each terminal will contain a furcation for the purpose of providing full duplex service. In addition, connectors will be required as shown in Figure 4-8;  $N$  will be required at the star node (one for

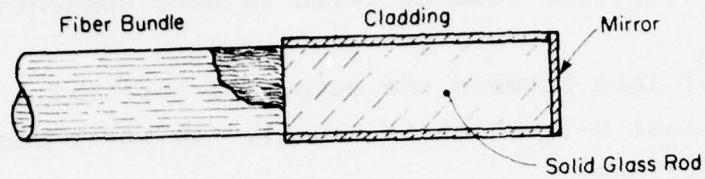


Figure 4-9. Star mixing rod.

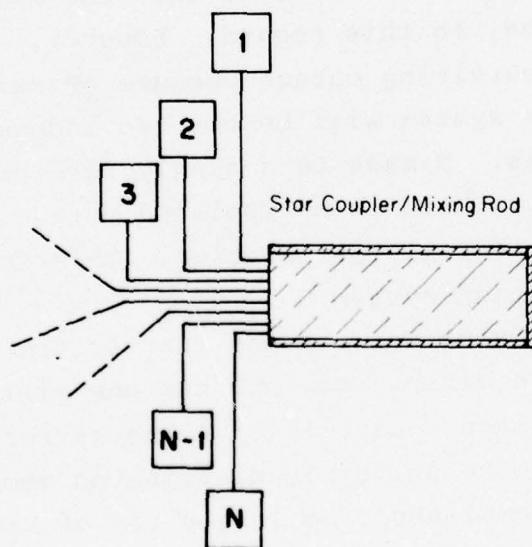


Figure 4-10. Star system configuration.

each terminal line) and a second at each terminal. Thus, there are two connectors in each line. The loss between any two terminals in the star system is

$$L_{\text{star}} = 4L_c + L_{\text{is}} + L_f + L_s , \quad (4-6)$$

where  $L_{\text{is}}$  is the insertion loss of the star coupler,  $L_f$  is as defined in the previous section and  $L_s$  (star loss) is the counterpart of  $L_{\text{PF}}$  for the tee system.  $L_s$  includes not only the packing-fraction loss, but also that loss attributed to the fact that  $N$  terminals are connected to the coupler. Ignoring the packing-fraction loss,

$$L_s = 10 \log (1/N) . \quad (4-7)$$

Actually, less than  $1/N$  of the available power is coupled to each terminal because of the packing fraction.

The star system is, unfortunately, vulnerable to total system failure. If the star coupler fails, all terminals are isolated; communication capability is then lost. This disadvantage tends to diminish the luster of the other advantages it enjoys.

#### 4.2.2 Comparison of the losses of the star and the tee systems

Equations (4-5) and (4-6) ignore the input-output loss at each terminal; the equations also neglect the attenuation of the fiber waveguide. The input-output losses will be the same for the two systems; hence, they do not enter into a comparison. Waveguide attenuation may introduce differences since the lengths of cable are different in the two systems. However, for systems of practical interest, distances will be moderate and the losses described above will dominate.

For the purpose of comparison, we will concentrate on the losses which depend on  $N$  and we consider only  $N \geq 4$ . The justification for such restriction is as follows: for  $N < 3$ , the

differences between the star and tee systems will be small. The total loss will be small and fiber attenuation may become important; such loss was ignored in this analysis. For only a few terminals, system architecture will be dictated more by system needs than by loss. The vulnerability to total failure, for example, may then take on more importance since loss of the two systems (star and tee) will likely be within a few dB of each other (see fig. 4-11). As  $N$  increases beyond 4, however, the variation with  $N$  becomes more important.

Figure 4-11 is a plot of those losses that depend on  $N$  for each of the two systems,  $N \geq 4$ :

$$L(N) = \begin{cases} -6(N-3) \text{ dB} & \text{tee system,} \\ -10 \log (N) \text{ dB} & \text{star system.} \end{cases}$$

The first equation assumes the following:

$$\begin{aligned} L_C &= -0.75 \text{ dB,} \\ L_{PF} &= -2 \text{ dB,} \\ L_{it} &= -0.5 \text{ dB.} \end{aligned} \tag{4-8}$$

Notice that  $L(N)$  is approximately the same for the two systems for  $N=4$  when the losses of equation (4-8) are used.

The curves of Figure 4-11 should be used with some caution. Value has not been attached to some of the practical differences between the two. In particular, the heart of the star system is a single component: the star coupler. Graceful or catastrophic degradation of that component will have impact (graceful or catastrophic) on system performance.

#### 4.2.3 Reduction of reflection loss

It was mentioned earlier that Fresnel reflection is a fundamental cause of loss. The fraction of incident power reflected is

$$R^2 = \left( \frac{n_o - n_1}{n_o + n_1} \right)^2$$

in the case of unpolarized light at or near normal incidence.

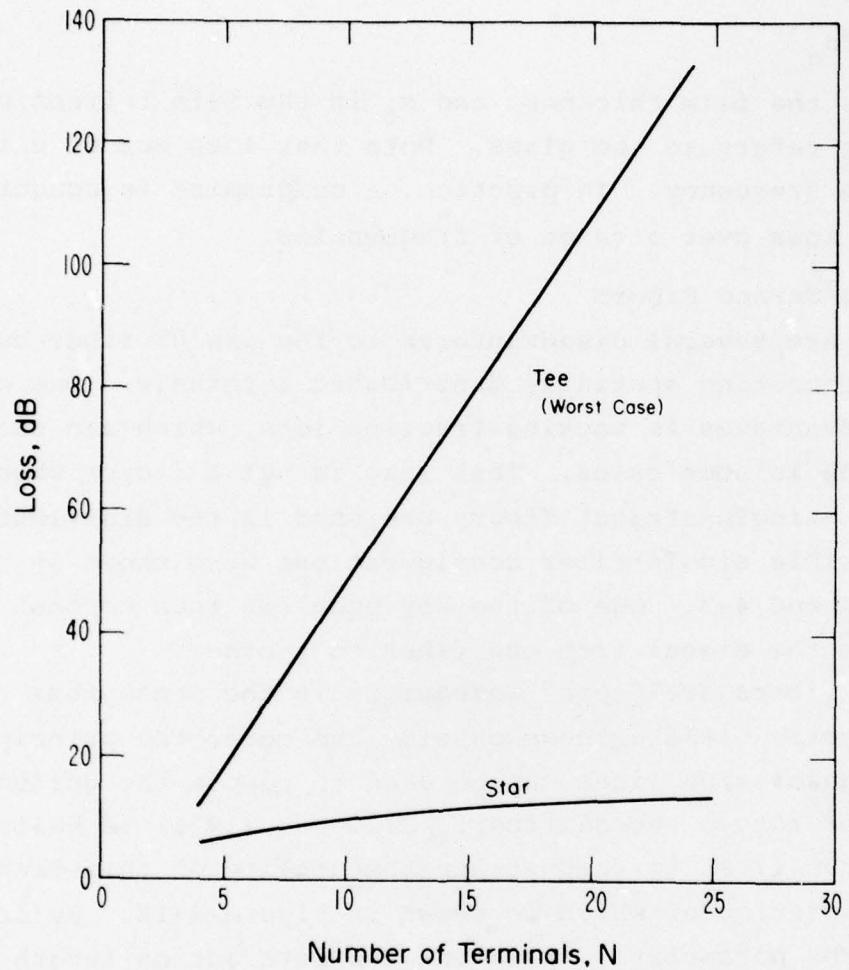


Figure 4-11. Data bus losses.

This loss can be reduced through the simple mechanism of coating with a thin transparent film. The film characteristics are adjusted so the light reflected from the air-film surface annuls the light reflected from the film-glass surface. This destructive interference is accomplished if the two reflected waves have equal amplitude and opposite phase. This occurs when

$$n_f t_f = \lambda/4$$

and

$$n_f = \sqrt{n_g}$$

where  $t_f$  is the film thickness and  $n_f$  is the film refractive index. Subscript g refers to the glass. Note that loss can be eliminated at only one frequency. In practice, a compromise is sought to reduce the loss over a range of frequencies.

#### 4.3 Single-Strand Fibers

There are several disadvantages to the use of fiber bundles for interconnecting spatially distributed terminals. One of those disadvantages is packing-fraction loss, which can become as high as 3 dB in some cases. That loss is not a factor when individual (single-strand) fibers are used in the distribution link. Possible single-fiber configurations were shown in Figures 4-2 and 4-3. One of the key problems then becomes that of coupling the signal from one fiber to another.

Since fibers are "open" waveguides in the sense that the electromagnetic field extends outside the core, the principles of coupled transmission lines can be used to couple the guided modes of one fiber into a second fiber. J.J. Pan (1976) is believed to have been the first to demonstrate the utility of this technique, the representation of which is shown in Figure 4-12. By proper choice of the parameters, including the interaction length, one can, in principle, adjust the coupling level, as desired. There is a tradeoff between the coupling length and the amount of bend introduced, although too much bending may cause increased insertion loss by increasing the radiation loss. The bend improves coupling by virtue of an apparent increase of phase velocity in the outer ring of the bend, due to the geometry of the bend and to a decrease in dielectric constant along the outer periphery. The increased phase velocity carries with it a decreased decay coefficient with resultant greater interaction between the two fibers. Coupling can further be enhanced by stripping off the fiber cladding in the coupling region. This further enhances the interaction by causing the field of fiber 1 to extend further into fiber 2.

The coupler pictured in Figure 4-12 is bi-directional. It introduces excess loss when  $P_3 + P_4 < P_1$  if coupling is from  $P_1$  into  $P_3$  and  $P_4$ . The excess loss is calculated from

$$10 \log \frac{P_3 + P_4}{P_1} .$$

Excess loss of 0.6 to 0.8 dB was reported by Pan (1976) but more than 1 dB was experienced by Barnoski and Friedrich (1976). The coupled power ratio ( $P_4/P_1$ ) can easily be adjusted to 10 dB with directivity of 21 dB reported.

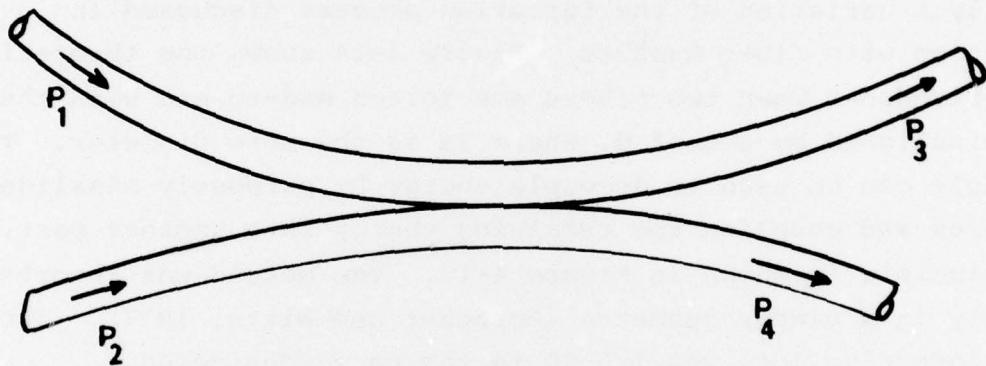


Figure 4-12. Coupled transmission lines.

A variation of this theme was recently reported, based on a fused section of two biconical tapers (see Figure 4-13).

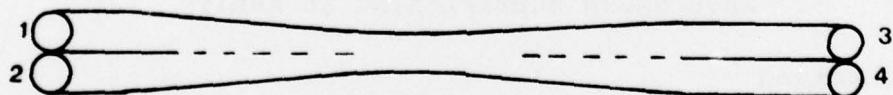


Figure 4-13. Biconical tapered couplers.

In this process (Kawasaki and Hill, 1977), two biconical tapers of multimode fibers are twisted around one another and put under spring tension. A microtorch flame is then used to fuse the two fibers while they are under tension. The tension causes them to elongate while being fused and the twist causes them to stay together. The resulting excess loss was only slightly greater than 0.1 dB in several cases.

Yet another method of coupling to and from single fibers is based on the fact that there is a power loss if two fibers are not axially aligned when they are joined end-to-end. This is actually a variation of the furcation process discussed in connection with fiber bundles. Figure 4-14 shows the theoretical loss introduced when two fibers are joined end-to-end with their axes misaligned by amount  $d$ , where  $2a$  is the core diameter. This principle can be used to decouple energy by purposely misaligning two cores and coupling the remaining energy into another port. The principle is shown in Figure 4-15. The method was reported recently in a planar geometry (Auracher and Witte, 1977). The total insertion loss was 1.5 dB in the case considered.

Yet another technique, which may be quite useful because of the simplicity and the control that is available, uses a slightly tapered section of multimode fiber. Coupling is accomplished through the side of the tapered section by illuminating the taper in well-defined angles (Barnoski and Morrison, 1976). A similar approach using a prism had been used earlier by Midwinter (1975).

## 5. WAVELENGTH MULTIPLEXING IN ARBITS

### 5.1 Introduction

It is conceivable that as a distribution system expands, some of the links in the network will eventually exceed the installed capacity. This might be the case in the loop network, for example, as terminals are added in an orderly growth pattern. It may also be the case in certain parts of a tee network if a prolific use is made of video. When capacity is reached, it is

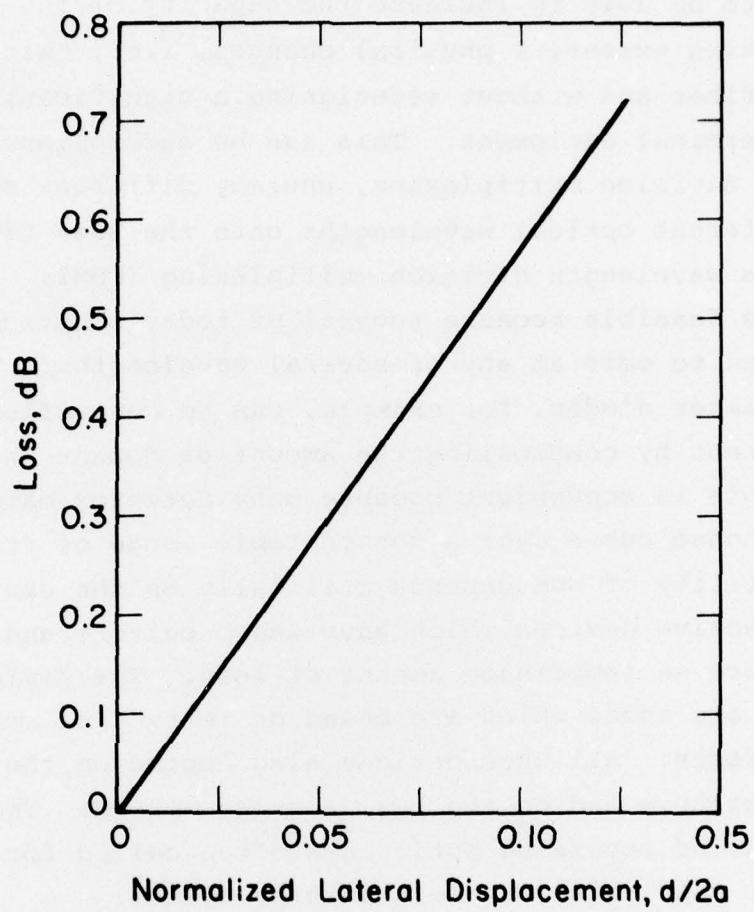


Figure 4-14. Loss due to lateral displacement of fibers.

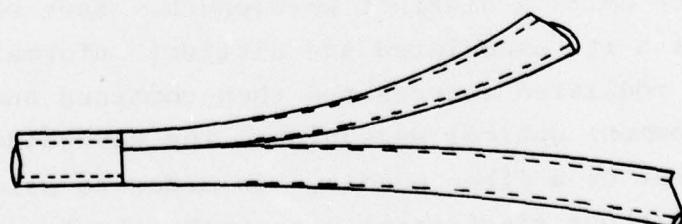


Figure 4-15. Coupling via misalignment.

desirable to be able to increase the capacity of the system without making extensive physical changes, i.e., without replacing fiber and without redesigning a significant portion of existing terminal equipment. This can be accomplished by using wavelength division multiplexing, whereby different signals are put on different optical wavelengths onto the same fiber. This is known as wavelength division multiplexing (WDM).

WDM is feasible because several of today's optical sources can be tuned to emit at any of several wavelengths. The semiconductor laser diodes, for example, can be controlled to a certain extent by controlling the amount of dopant in the active region. This is convenient because many detector materials have a flat response curve over a considerable range of frequencies.

The utility of WDM depends critically on the use of wavelength-selective devices which have sharp cut-off and which do not introduce an inordinate amount of loss. The devices having least loss are those which are based on refractive or interference effects. All such devices also depend on the orientation of the light beam and on the beam angular spread. Thus, collimating and focussing optics are often called for and this adds to the complexity of the resulting system.

### 5.2 Optical Filters for Use With WDM

WDM is accomplished by utilizing several optical sources, each of which emits a distinct wavelength. Each source is modulated with its associated and distinct information signal and the various modulated sources are then combined and coupled onto a single (common) optical waveguide. The waveguide can be either a single fiber or a fiber bundle. In order to allow channel separation without significant crosstalk, the frequency separation of the carriers must be several times the linewidth.

WDM is difficult owing to the requirements for suitable optical filters. The filters must have a narrow pass band, high rejection of neighboring channels, low loss, temperature stability, and electrical and mechanical simplicity. This is a stringent

list of requirements; however, WDM has distinct advantages if filtering can be accomplished economically.

If the detector in the receiver circuit has sufficiently narrowband characteristics, the need for filters is eliminated (at least in the receiver). In general, photodiodes have a broadband response; wavelength discrimination is therefore required as part of the detector circuit. Filters are either bandpass filters or dichroic filters. Dichroism is the property of exhibiting two colors; a dichroic substance will reflect one color (range of frequencies) and transmit another.

#### 5.2.1 Bandpass filters

Absorption filters normally exhibit bandpass characteristics, which bandpass is a function of the material comprising the filter. The transmissivity of the filter may reach as much as 50%; they are generally quite broadband. Unfortunately, the filter properties depend on temperature and humidity and, hence, they tend to exhibit unstable properties. However, even without that undesirable environmental dependence, the bandwidth of absorption filters is generally not acceptable and the rejection properties are not sufficiently sharp; i.e., the skirt of the transmissivity curve as a function of wavelength is not steep enough.

A more desirable bandpass filter can be made by using several layers of dielectric material. By allowing each layer to have a different refractive index and by increasing the number of layers, the bandwidth of the filter can be decreased. The filters operate on an interference principle, yielding constructive and destructive interference which yields the bandpass characteristics. The cut-on and cut-off properties are suitably sharp; transmittance of 50% or more in the pass band is quite common. Once manufactured, they can be hermetically sealed to provide isolation from the environment. The bandwidth can be adjusted from about 0.5 nm to 50 nm by adjusting the structure of the filter.

Since the optical thickness of the film is a function of the angle of incidence, the interference properties, and hence the filtering properties, will depend on the degree of collimation of incident light. At the output of a fiber bundle, for example, the light beam is often diverging; the pass band of the filter then becomes correspondingly wider.

If the refractive index of the material comprising the filter is a function of temperature, which is usually the case, the characteristics of the filter will change with temperature. The center wavelength shift is approximately a linear function of temperature. In addition, the transmittance can be expected to suffer as temperature increases. The sensitivity to temperature variation can be reduced by using only certain combinations of dielectric materials. The price paid for this luxury is usually an increased sensitivity to incidence angle.

### 5.2.2 Dichroic filters

A dichroic filter is capable of transmitting all frequencies above a certain cut-off frequency and reflecting all lower frequencies. The filter is thus a high-pass or a low-pass filter, depending on how it is used. It is an all-dielectric device which normally requires an angle of incidence of  $45^\circ$ . The efficiency of such filters is quite good, being greater than 80% in most cases; thus, a reasonable insertion loss is offered with dichroic filters. Being dielectric filters, they are somewhat dependent on the environmental conditions, as discussed for the interference filters, above.

A configuration for coupling light from four sources at different wavelengths into a fiber is shown in Figure 5-1. Figure 5-2 shows the high-pass characteristics required for such a configuration to work. The serial filters are selected and positioned according to their frequency characteristics. In this configuration,  $\lambda_4 > \lambda_3 > \lambda_2 > \lambda_1$ . Each filter has a lower cut-on wavelength than the preceding one. A similar arrangement can be used at the receiver if broadband photodiodes are used.

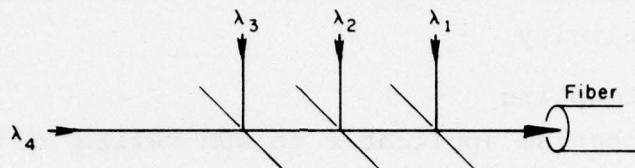


Figure 5-1. A method of WDM.

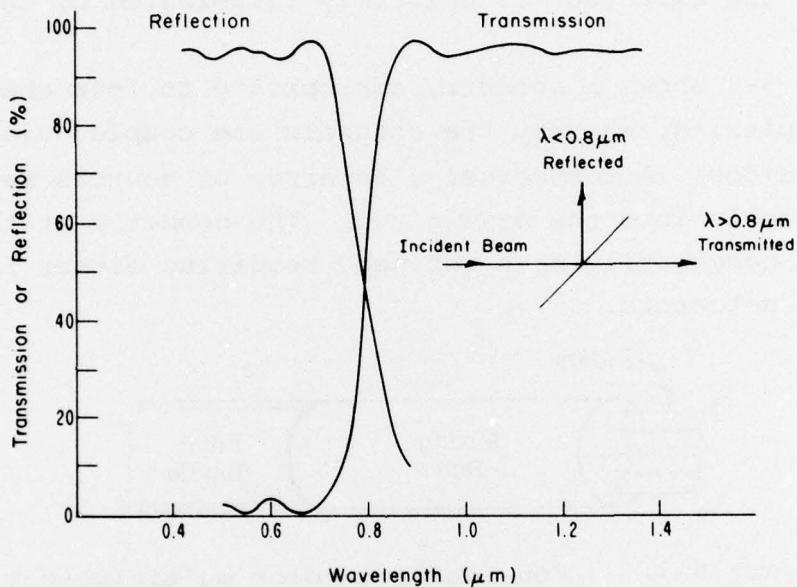


Figure 5-2. Filter characteristics for use with WDM.

The disadvantage of such an arrangement is in the complexity and cost of the apparatus. A reasonably good alignment of components is required. Furthermore, the cost of the filter depends on the nature of the cut-on, cut-off characteristics. If the cut-off extends over only a few nanometers, the cost is currently quite high; yet, this is required for efficient operation of such a system architecture.

Another disadvantage stems from the geometry of the arrangement. There is a space loss and beam divergence loss because of the distance between the sources and the fibers. Some of these

losses can be overcome through lenses, but such lenses detract from system simplicity.

### 5.3 Furcation Coupling

Another mechanism applicable to WDM relies on the use of a mixing rod; such a rod spatially mixes the input from several channels so that at the exit from the rod, the signal is a mixture of the signals from each separate channel. The mixing is accomplished through a multitude of reflections at the rod boundaries; the exit face is uniformly illuminated by the various channels.

Figure 5-3 shows a geometry appropriate to four channel color multiplexing, whereby the channels are coupled via separate fiber waveguides. Alternatively, an array of sources may be coupled directly into the mixing rod. The geometry at the receiver is necessarily more refined, requiring either filters or narrow band detectors.

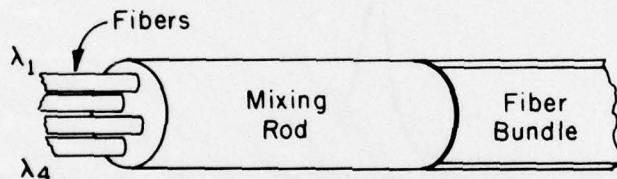


Figure 5-3. Four channel color multiplexing.

## 6. SUMMARY

We have examined some of the key features associated with the use of optical waveguides in ARBITS. There are obvious advantages of fibers over coaxial cable, not the least of which is bandwidth. While current cost of fiber waveguide is greater than most coaxial cables, the bandwidth capability actually renders the fiber competitive. This is due to the fact that coaxial cable size must increase as data rate increases; this is not the case for fibers, although the quality of fiber called for is a function of data rate. These factors are accounted for in the model presented in Section 2. Using that model, it was shown

that the cost of fiber systems is actually less than that of coaxial cable systems when data rate becomes moderately high.

The use of single strands of fiber in ARBITS shows considerable promise now that mechanisms for coupling to such fibers are emerging. Some new and clever techniques show distinct promise but they are still laboratory devices. As such devices become more readily available, they will have significant impact on ARBITS.

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